

ELEMENTS
OF
NATURAL PHILOSOPHY.

By GEORGE MILLER, D. D.
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TO THE
REV. RICHARD MURRAY, D. D.
P R O V O S T
O F
TRINITY COLLEGE, DUBLIN,

THE FOLLOWING TREATISE,
DESIGNED FOR THE IMPROVEMENT OF
THE STUDENTS WHOSE EDUCATION
HE SUPERINTENDS,

IS RESPECTFULLY INSCRIBED

BY HIS MOST OBEDIENT,
AND VERY HUMBLE SERVANT,

GEORGE MILLER.

March 11, 1799.

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PART I.

INTRODUCTION.

NATURAL Philosophy is that science, which explains the appearances of the material system, by pointing out the general rules according to which they are produced, and the material agents which are immediately concerned in their production. Of the intimate or essential nature of matter we are, and must ever continue, wholly ignorant. Our knowledge of it must be derived altogether from observation, and can therefore extend no farther than those impressions which it is fitted to make upon our senses, and which are denominated its properties. This ignorance of the essential nature of matter must necessarily preclude us from any acquaintance with the nature of the connexion of causes and effects. All which we are able to discover, is the mere fact of that connexion. We may observe that a certain

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event,

event, or the presence of a certain agent applied in a certain manner, always precedes the occurrence of some other event, and thence conclude, that the latter somehow derives its existence from the former; but the mode in which the one gives existence to the other, is not cognizable by the senses, and is therefore not within the limit of our knowledge of the operations of nature. We know, for example, that two magnets, placed near each other in a certain manner, will be attracted towards each other; and, if either of them be turned into the opposite direction, will be repelled from each other. The uniform connection of these results with the preceding circumstances, induces us to ascribe them to those circumstances as to their causes; but we are wholly ignorant of the mode in which the magnetic efficacy produces these effects. Even though we could ascertain the existence of that magnetic fluid, which has been supposed for the explanation of these phenomena, we should not have approached nearer to the knowledge of the nature of causation. It would still be impossible to conceive, what is that exertion of power, which could repel the particles of this fluid from each other, and could attract them towards the particles of the magnets. We should indeed have discovered a more comprehensive analogy of the facts of natural operations, but should continue equally ignorant of the efficacious mode of their production. Experience is the only source of physical knowledge, and it can give us information only of the impressions made on our senses, and of the various combinations in which these impressions may be observed.

It

It was indeed generally supposed before the time of the celebrated Bacon, that the human mind was able, with but a cursory and superficial view of the operations of nature, to invent such explications as would fully correspond to the appearances. But he sufficiently exposed the futility of this ancient method of philosophizing. * Its defects he reduced to three heads. Principles were assumed at pleasure, if they could in any degree contribute to furnish a solution of a difficulty; the prepossessions of the understanding itself were revered as unquestionable truths; and the immediate information of the senses, unaided by experiments, was considered as decisive evidence. The small progress which philosophy had made, was indeed an abundant proof that there was something materially erroneous in the manner of proceeding. After the lapse of so many ages the knowledge of nature had advanced little beyond those simple and obvious discoveries, which lay open to ordinary experience. But the superior mind of Bacon surveyed the whole field of human knowledge, and pointed out the true mode in which we might hope to extend our information. To reduce every principle to the rigorous test of accurate experiment, was pronounced by him to be the only effectual method of penetrating into the secrets of nature; and in the short period of two centuries a strict observance of the rules which he proposed has been productive of the happiest effects.

But though the mind is thus limited in its physical researches to that information, which the senses may by

* *Instauratio magna. Distrib. Operis.*

well-contrived experiments be enabled to supply; and is incapable of apprehending either the essential nature of the bodies which it examines, or the mode in which the Creator has connected with each other the causes and effects of natural operations: still the contemplation of the material system, and the investigation of the laws by which it is regulated, are highly deserving of attention. To discover that the phenomena of this vast and complicated universe are subjected to a few simple and general rules, and that the variety and intricacy, which perplex our understandings, are but the result of the infinitely varied circumstances in which the operations of nature are performed, must naturally impress us with the deepest sense of the perfection of that Great Being, on whose will all these appearances depend. This impression must indeed be strengthened by the very limitation of this science. When, amidst all our experimental discoveries of the processes of nature, we cannot any where perceive the efficacious principle, which connects with each other causes and effects; when we perceive only a methodized arrangement of natural appearances, simple indeed and beautiful, but indicating no necessary and operative production of them; we must entertain a still more lively conception of that superintending power, which either continually directs the minutest parts of this great and intricate machine, or originally bestowed on them that incomprehensible active efficacy which maintains its movements. The practical application of the discoveries of philosophy to the uses of life might perhaps indeed be improved, if we were capable of perceiving the mode in which each

each phænomenon is connected with that, which is denominated its cause; since the inductions of experience might then be generalized with greater certainty: but in those cases in which we are able to institute a variety of accurate experiments, conclusions may be formed with so strong an evidence, as to ensure the production of a desired effect whensoever those agents shall have been applied, with which it so uniformly and so plainly appears to be in some manner connected.

The modern method of philosophizing, though it professes to investigate by a careful observation of nature the varieties and the laws of natural appearances, does not however altogether reject the aid of hypotheses. They must not indeed be formed arbitrarily, and with but a superficial view of the phænomena, which it is intended that they should explain; but when a number of accurate observations have rendered it highly probable that, agreeably to the general connection of the operations of nature, a certain agent has contributed to the production of the effects which we are examining, to adopt such a supposition may be useful by directing us to farther trials, which shall either verify or refute it. Thus, when Franklin observed that the phænomena of electricity considerably resembled the effects of lightning, he concluded that it was highly probable that they were produced by causes perfectly similar. He did not however indolently acquiesce in the probability of this hypothesis, but contrived by his electrical kite to bring down from the clouds the electrical fluid, and, by subjecting it to similar experiments, to demonstrate its identity with that

that which had been discovered to exist in our immediate neighbourhood. An hypothesis must be adopted only upon trial. It must afterwards be subjected to a rigorous examination by experiment, and the ingenuity of the experimenter consists in devising modes of observation, by which the operation of the supposed cause may be unequivocally ascertained. The illustrious Bacon has well described this kind of experiments. To the ancient simple kind of induction he objects, that it was frivolous and inconclusive, and requires that a more discriminating and cogent form of it should be adopted. **Ea enim, de quâ dialectici loquuntur, quæ procedit per enumerationem simplicem, puerile quiddam est, et precario concludit, et periculo ab instantiâ contradictoriâ exponitur, et consueta tantum intuetur, nec exitum reperit. Atqui opus est ad scientias inductionis formâ tali, quæ experientiam solvat, et separet, et per exclusiones ac rejectiones debitas necessario concludat.*

Before that we proceed to examine the operations of nature, it is necessary to state those simple and general principles, which are called the Laws of Motion. Their truth may be established by easy and obvious observations, and receives confirmation from the most advanced researches into the movements, by which the phænomena are produced.

Law 1. Every body perseveres in its state, either of rest, or of uniform rectilinear motion, unless it be by some force impressed upon it compelled to change its state.

* *Instauratio magna. Distrib. Operis.*

That

That a quiescent body continues in its state of rest until the operation of some external force compels it to change that state, and begin to move, is obvious to general experience. But that a body in motion will in like manner continue to move forward with the same unvaried velocity, was not equally easy to be apprehended; and accordingly even Kepler thought that a continued exertion of force was requisite for maintaining the motion of a body. The motions produced on earth do indeed become languid and cease, but we can assign the external causes of their diminution and cessation, namely the resistance of the medium through which they move, or the friction of some contiguous surface; and as those causes are diminished the motions are continued longer. Bodies projected near the surface of the earth are soon perceived to deviate from a rectilinear course, and to fall towards the earth; but their weight is a force perpetually drawing them from their original direction. Mr. Atwood has contrived an ingenious experimental confirmation of this law of motion. * He directs that two equal weights should be balanced by a fine flexible line, which passes over a pulley. The weight of each body being counterbalanced, they will remain quiescent until some force shall be applied. But if any impulse be communicated to either of them in a vertical direction, it will descend perpendicularly through equal spaces in equal times, that is, it will move downwards with an uniform and rectilinear motion. The instrument, by which this

* Analysis of a Course of Lectures on the Principles of Natural Philosophy, Article 39.

experiment is performed, is represented in plate 1. fig. 1. The two boxes *A* and *B*, of equal and similar dimensions, and containing equal weights, are suspended in equilibrium. *A* is then raised above the ring which is attached to the instrument, and laden with the oblong weight, which is in the figure represented as resting on the ring. When the box *A* descends through the ring, this additional weight is removed by the ring, and the motion of *A* from the ring downwards will be the effect of the impulse which it had communicated. If now the pendulum connected with the instrument be let go at the same instant at which the additional weight is removed, it will be found, by observing the motion of the pendulum and the scale marked on the instrument, that the box *A* will descend through equal spaces in equal times, that is, that it will move uniformly downwards.

Law 2. Every motion, or change of motion, is proportional to the force impressed, and is made in the same direction in which that force is impressed.

This may be considered as a consequence of the preceding, since a body is unable to make any change in its own state. It may also be confirmed by experiment. Whilst the sum of the weights of the two bodies remains the same, let it be variously divided between them, so that the differences of weight, which are the moving forces, may be varied; and let these differences consist of oblong weights imposed on the box *A*. The box *A*, thus variously laden, will be found to descend to the ring in equal times through spaces proportional to the forces; and, when these

these forces have been removed, to move uniformly (law 1.), with the motions acquired in those equal times, through other spaces also in the same proportion.

The whole quantity of matter moved consists of the sum of the bodies *A* and *B* and of the line by which they are suspended*. If the weight of the last were sensible, it would produce an inaccuracy in the experiment in consequence of the variable lengths of its two parts. This however is imperceptible. The effect of the resistance of the air is also insensible, as the resistance which it gives to bodies moving with small velocities is inconsiderable. No other difficulty therefore remains except the friction of the wheel, over which the line suspending the boxes is passed. This however is diminished by causing the axle of the wheel to rest on other wheels, called friction-wheels, whose use will be explained hereafter.

Cor. 1. To determine the direction and velocity of the motion caused by the combination of two forces acting in oblique directions.

If the body would in a certain time be driven by the impulse of one force from *A* to *B* (plate 1. fig. 2.), and in the same time by the impulse of the other from *A* to *C*; complete the parallelogram *ABDC*, and the combined impulses will cause the body to move uniformly in the same time from *A* to *D*. The motion will therefore be perform-

* In a strict consideration of the experiment the motion communicated to the wheel should also be included.

ed in the direction of AD , and its velocity proportioned to AD . For the force acting in the direction AB parallel to CD , will not affect the velocity with which the body approaches towards the line CD . Nor will the force acting in the direction AC parallel to BD , affect the velocity with which the body approaches towards the line BD . The body will therefore arrive at the line CD in the same time, whether the latter force act or not; and at the line BD , whether the former act or not. Consequently at the end of the given time it will be at the intersection of these lines, that is at the point D ; and, by the preceding law, it will have moved with an uniform and rectilinear motion. It will therefore have moved uniformly along AD in the same time, in which with the impulse of either force separately impressed it would have moved uniformly along AB or AC ; and its velocity is accordingly proportional to the line AD .

The velocity communicated by the combined forces is, when their directions are oblique, less than the sum of the velocities which they would separately communicate; because the diagonal AD , to which it is proportional, is (Euc. lib. 1. pr. 20.) less than the sum of AB and BD , or of AB and AC , to which the velocities separately communicated are proportional.

This, which is called the Composition of Motion, is illustrated by the instrument represented in plate 1. fig. 3. $ABCD$ is a wooden square, so contrived that the corresponding square $BEFC$ may be drawn out from it, as in the figure. To the latter is joined a pulley, which will be at H when it is pushed in, and at h when it is drawn out. Let also a wire k be fastened to it, so as that

that it may move with it under the pulley; and let a ball G be made to slide upon the wire. When the moveable square is pushed in, the ball may be drawn along the wire by the thread m passing over the pulley H ; or by drawing out the square it may be moved with the wire in a direction parallel to DC . If now the extremity of the thread be fastened at I , so that the ball may have both motions impressed on it together; it will, whilst the square is drawn out, be observed to move along the diagonal of the fixed square to the point g .

In the case which has been considered, it is not necessary to suppose, that the impulses of the two forces are impressed at the same instant; since, by the preceding Law of Motion, the motion communicated by either will continue unvaried, and may afterwards be combined with any subsequent impulse.

*Cor. 2. If one of the forces communicate but a single impulse to the body, and the other act continually during the time of the motion, the path of the body will be a * curve line.*

If the body A , fig. 4. plate 1. be impelled in the direction Ab with a force which would cause it to move to b in an indefinitely small portion of time, and in the direction Ac with a force which would in an equal time

* In general, a curve line will be described, 1. when a single impulse is combined with a continued force; 2. when two continued forces, varying in different ratios, are combined together; 3. when an unvaried or constant force is combined with a varying one.

cause it to move to *c*, it will at the end of that small portion of time have moved from *A* to *d*. This motion continued uniformly and in the same direction, by the preceding Law, would cause the body to proceed in the next equal portion of time through *de* equal to *Ad*; but another impulse of the continued force in the direction *df* will cause it to move in the diagonal *dg*: and in the like manner in the third instant it will move through *gk*: and so on. But as the action of the force in the directions *Ac*, *df*, &c. is supposed to be continual, the change of the direction of the body will be continual, and its path a curve line.

*Cor. 3. If three forces, acting upon a body in directions * oblique to each other, be to each other in the ratios of the sides of a triangle drawn parallel to these directions, they will be in equilibrium, and the body will remain at rest.*

This is proved by the composition of forces; since it may be shewn, that any two of the three forces will compound a force equal and contrary to the third. Thus, if three forces drawing the body *A*, fig. 5. plate 1. in

* If two of the forces act in directions exactly opposite to each other, the three forces will not be in equilibrium; for the two forces directly opposed to each other will not compound any intermediate force to counterbalance the third given force: nor can this compound with either of the other given forces one which shall be directly opposed to the remaining one. Hence it follows, that a cord not perpendicular to the horizon cannot be stretched, so as to be perfectly straight: for if this could be done, the tending forces would be directly opposed to each other, and yet counterbalance the weight of the cord, which is a third force acting in a direction oblique to those of the former.

the

the directions AB , AC , and AG , be one to another in the ratios of the sides of the triangle ABD , which are respectively parallel to their directions; complete the parallelogram $ABDE$, and AC will be proportional to the force which acts upon the body in that direction, because (Eucl. book 1. pr. 34.) it is equal to BD , which is proportional to the same force by the hypothesis. But, by Cor. 1. the two forces, which are proportional to the forces acting in the directions AC and AB , compound a force proportional to the diagonal AD , and in the direction of AD . Since then the third force, which pulls from A to G , is, by the hypothesis, proportional to the same line AD considered as a side of the triangle, but acts in the opposite direction, the action of the compounded force will be equal and contrary to that of the third force, and the body will remain at rest.

This corollary may be proved by experiment, if three cords fastened to the body be passed over pulleys placed on a plane parallel to the horizon, so that the cords may respectively be coincident with, or parallel to the sides of a triangle drawn upon the plane; and weights be attached to the extremities of the cords, which shall be one to another in the ratios of the sides of the triangle respectively coincident with, or parallel to the cords, by which they are suspended.

This experimental confirmation of this corollary, is likewise a confirmation of the composition of motion explained in corollary 1. Since it proves by experiment

that

that the force compounded of any two of the three given forces is such, as was there described.

Conversely, *If a body, on which three forces act in directions oblique to each other, be at rest; these three forces must be proportional to the sides of any triangle, which are respectively parallel to their directions.*

For if they were proportional to the sides of such a triangle, they would balance each other, by what has been already proved. Therefore, if any force were greater or less in regard to the other forces than in the ratios of the sides of the triangle respectively parallel to their directions, the equilibrium would be disturbed, and the body could not remain at rest, contrary to the hypothesis*.

* On this composition of forces or motions is founded what is called their Resolution. By the resolution of forces is meant the investigation of two distinct forces, which by their combined operation would produce the same effect as any given single force. This is important in Mechanics, whenever the direction, in which any

* The equilibrium of forces acting in oblique directions is not confined to the case of three, but may be expressed generally in the following manner: If any number of forces acting in directions oblique to each other, be to each other in the ratios of the sides of a right-lined figure drawn parallel to their directions, there will be an equilibrium. For these may be successively combined into forces represented by diagonals of the figure, until the case shall be at length reduced to that of three forces proportional to the sides of a triangle respectively parallel to their directions.

force

force acts, is oblique to the direction in which the effect is to be produced; for, by resolving the force into two forces, it is determined how much force is really exerted to the production of the effect, and how much is lost by the obliquity of the application of the given force.

Cor. 4. *Any force acting upon any object being given, to find what force is exerted upon that object in any proposed direction oblique to that in which the given force is applied.*

In plate 1. fig. 6. let a force act upon the body CD , which in a certain time would cause the part a to which it is applied to move from a to b ; but let the body be fixed at C , so that the point a can only move in a direction ae perpendicular to CD , whilst it describes an arch round the point C . From b draw a right line bd perpendicular to ae ; and, if ab represent the given force, ad will be proportional to that force which really acts to move the point a . For complete the parallelogram $adbg$, and the forces represented by ad and ag would together produce the same effect as the given force represented by ab (cor. 1.): but the force ag , which acts in the direction of CD , can not have any effect in moving the body round the point C . Therefore the other force ad , which acts in the direction of the circular motion round C , can alone operate in moving the body.

In the same manner a given force may be resolved into two, whose directions shall form an oblique angle.

The resolution of motion may be confirmed by experiment. To a body circumstanced as CD , let a cord

be

be attached at *a* in the direction *ab*, and parallel to the horizon, and by this cord, passed over a pulley, let a weight be suspended: and if another cord in the direction *da*, and also parallel to the horizon, be attached likewise to *a*, and passing over a pulley sustain a weight, which shall have the same proportion to the former which *ad* has to *ab*, there will be an equilibrium.

Law. 3. Reaction is always equal and contrary to Action: or the actions of two bodies on each other are always equal, and exerted in opposite directions.

That a reaction is exerted in a direction contrary to that of every action, appears from many considerations. If the hand press against an obstacle, it experiences a similar pressure. If a horse draw a load, the retardation of his progress is the effect of the reaction of the load. But that reaction is equal to action, as well as contrary to it, is proved by experiments on the attractions of bodies. If two similar and equal bodies of the same kind of substance be set to float on water, and a cord attached to one be passed over a pulley connected with the other, and sustain a weight, which in its descent may cause an attraction to take place between the floating bodies; they will be equally drawn, and meet in the middle point of their original distance. In the like manner, if two magnets be so placed on pieces of cork floating on water, that an attraction may take place, each will attract the other; and, if their weights be equal, and the corks equal and similar, they will meet in the middle point of their original distance. In all cases of attraction,

tion, however unequal or dissimilar the bodies may be, it will be evident that the forces with which they meet are equal, since they will be observed to stop when they meet. Their velocities will not indeed be equal, except they are of equal weights and of similar and equal forms; and, therefore, they will only in this case meet in the middle point; but in all cases they meet with equal force, since the force of each is just sufficient to counterbalance that of the other.

Schol. 1. From these Laws of Motion is inferred, what has been called the Inertness of Matter, or the *Vis Inertiae*, which has been defined to be the power of resisting, by which every body perseveres in its state, either of rest or of moving uniformly in a straight line. That a body will persevere in its state, either of rest, or of uniform rectilinear motion, unless compelled to change it; is the 1st Law of Motion: and from the 3rd Law it appears that in yielding, it exerts an equal and opposite force upon that body, which compels it to change its state. This resistance to the operation of external force is therefore conceived to be a general property inherent in all matter, and is on that account also denominated *Vis Inertiae*. The *vis inertiae* of any body must be proportional to the quantity of matter contained in it, because, since it is a general property of matter, the inertness of the whole must be the aggregate of the inertness of all its parts.

Schol. 2. On account of this inertness of matter the momentum of a body, or the force with which it moves, must

must be distinguished from its velocity. If bodies were not inert, and consequently yielded without resistance to any force impressed, the moving forces need not have any reference to the masses, or quantities of matter, to be moved. But since all matter is inert, a greater force must be required for communicating a certain velocity to a greater quantity of matter.

The moving forces acting upon bodies, and the quantities of motion communicated to them in a given time, are proportional to the quantities of matter moved, and the velocities communicated, jointly.

For (by Law 2.) if the quantities of matter in the two bodies be equal, the moving forces will be as the velocities communicated. If, on the other hand, the velocities communicated be equal, the moving forces must be proportional to the quantities of matter moved with the given velocity; since a double or triple force would be required for communicating a determinate velocity to a double or triple quantity of matter. If, therefore, both the quantities of matter and the velocities are unequal, the moving forces, and consequently the momenta which they communicate in a given time, must be proportional to the quantities of matter and velocities, jointly.

Uniform velocity is that with which a body in any equal times moves through equal spaces.

Two different uniform velocities are proportional to the spaces, through which bodies, moving with these velocities respectively, would move in equal times.

If

If the velocities, with which bodies move, be variable, they can be compared together only by reducing them to the case of uniform velocities; that is, by comparing the spaces, through which bodies would move in equal times with the velocities continued uniformly from the instants of time, for which they are to be estimated. Such a comparison will enable us to determine justly the ratio of the two velocities at those instants; whereas a comparison of the spaces actually described would be affected by the subsequent variations of the velocities.

Such are the general principles which relate to the motions of bodies.

“If it be enquired,” * says Mr. Atwood, “from what evidence our assent to these axioms is derived, it may be replied, the evidences are of various kinds.

1. † From the constant observation of our senses, which tend to suggest the truth of them in the ordinary motion of bodies, as far as the experience of mankind extends,
2. From ‡ experiments properly so called. In these it always appears, that when the most effectual means are

* Treatise on the Rectilinear Motion and Rotation of Bodies. Sect. 9.

† “This argument depends not on the accuracy of these common observations, but on their extent; nothing contrary to the axioms being observed in the great variety of motions which are the daily objects of the senses.”

‡ “In experiments every power is removed, as far as may be, except those which are the objects of examination.”

used in order to subject these axioms, or laws of motion, to the severest and most minute examinations, by removing all impediments which may tend to occasion any deviation from the truth; the result is a more near agreement between the experiments and the axioms just mentioned, the more perfectly the intentions of the experiments are accomplished. This will lead to a belief, that if it were possible to remove every imperfection from the construction of experiments, and to encrease the acuteness of the senses, of which they are the objects, in an infinite degree, a mathematical coincidence between such experiments and the laws of motion would be observed.

3. From arguments a posteriori. Let a proposition be assumed as true even without evidence of any kind. If, by strict and logical reasoning, various conclusions are deduced, which upon examination are found consistent among themselves, and with experience, this will be a presumptive proof in favour of the principle assumed; and our assent to it will be the more strongly enforced, in proportion as the conclusions inferred, and the comparison of them with experience, have been more extensive. From the Newtonian axioms assumed as true, a system has been deduced and compared with phænomena in numberless cases: it has been applied to the motion of the planets and comets, to that of bodies on the earth's surface; even to the motion of those minute particles which compose both solid and fluid substances. A perfect agreement between these consequences of the axioms and matter of fact has been the result, no one instance excepted. These, and other similar arguments, upon the whole amounting to evidence scarcely inferior to mathematical

mathematical demonstration, are the grounds on which the laws of motion, although incapable of direct proof, are received as axioms, from which the various theorems concerning the effects of forces are systematically demonstrated."

In deducing the effects of forces from the general laws of motion, it must be obvious that considerable advantage must be derived from the application of mathematical reasoning, whenever it shall be found practicable. This, however, cannot be done directly. Abstract quantity alone is the object of Mathematicks. Extension is the object of that part of mathematical science, which is denominated Geometry; and number of that other, which is called Arithmetick. But Natural Philosophy treats not only of the spaces in which motions are performed, but of the times in which these spaces are described, of the velocities of the moving bodies, and of the forces from which their motions are derived. If mathematical reasoning can be applied in such investigations, it must be done by substituting for these physical quantities mathematical quantities, either right lines or numbers, which shall represent them, and applying to the former those conclusions which shall be proved mathematically to belong to the latter. This is allowable, because in Natural Philosophy we consider, not the essential nature of the subjects which we contemplate, but merely the proportion of the observable circumstances of the causes and effects which are concerned in natural operations. These proportions may be represented by
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the proportions of mathematical quantities, and the conclusions deduced from these by mathematical reasoning must consequently be applicable to those which they represent,

CHAP. I.

OF GRAVITY.

SECT. I. *Of Gravity near the Surface of the Earth.*

THE most obvious phenomenon which we can discover in our own, and in surrounding bodies, is that tendency towards the earth, which is called weight or Gravity. The far greater part of all the bodies which we behold, are evidently influenced by this tendency. If they are let go, they fall to the earth in right lines; if they are projected in directions oblique to the horizon, they descend in curve lines, in consequence of the combination of the velocities with which they are projected and their tendency to the earth, as has been explained in Cor. 2. of the 2^d law of motion; and if they are projected perpendicularly upwards, after a short time their motion is destroyed, and they return to the earth. We ourselves experience in ourselves the necessity of some support for counteracting this tendency. That the air which surrounds the earth is heavy, is proved in that
part

part of Natural Philosophy which is called Pneumatics, and its weight is measured by the barometer. Even the smoke which ascends in air, is raised from the earth only because it is less heavy than the air through which it ascends, as wood will rise to the surface of water. This is shewn by a very easy experiment. The smoke of a candle will ascend to the top of a tall receiver when full of air; but if the air be exhausted, it will descend to the bottom. The only substance, whose actual gravity has not yet been discovered, is the hypothetical matter of heat, denominated Caloric by the Chemists; it being now with good reason supposed, that the phenomena of heat and cold are occasioned by the emission or absorption of a fluid substance. As far as experiments have yet been made, bodies seem to suffer a diminution of weight, when heated.*

† Some have supposed that the inertness of matter, explained in the Introduction, is the same with this force, or rather its effect; but that this opinion is erroneous, will appear from considering that the inertness of bodies may be proved to exist in cases, in which their weights are counterbalanced, and therefore produce no effect. Thus, in Mr. Atwood's instrument, plate 1. fig. 1. if different weights be successively placed in the box A,

* Dr. George Fordyce made the experiment by weighing the same quantity of water (about 1700 grains) when frozen and when unfrozen, at the temperature of 32° , in a room where the air was of the same temperature. The ice was near one-fiftieth of a grain heavier. Phil. Transf. Vol. 75, p. 362.

† See the Cyclopædia, Art. Mechanics.

and

and balanced by equal weights in the box *B*, the gravity of the boxes is destroyed: but it will be found, that the same force, communicated by the oblong weight imposed on the box *A*, will cause it to descend with less velocity when it contains a heavier weight; that is, the greater quantity of matter will oppose a greater degree of inertness to the operation of the moving force, and therefore will acquire from it a less velocity.

The phenomena therefore of falling bodies are the effects of a distinct force, denominated Gravity, which draws them towards the earth in directions perpendicular to the horizon, and consequently, as the earth is nearly spherical, in directions tending nearly towards the center of the earth. It is the object of this chapter to explain and establish the laws which regulate its operations.

Lemma. All bodies, falling near the surface of the earth, descend with equal velocities when the resistance of the air is removed.

If a feather and a piece of gold be let fall at the same instant from the top of a tall exhausted receiver, they will descend to the bottom together.

Law 1. The gravities of bodies near the surface of the earth are proportional to their quantities of matter.

For it has been shewn, in schol. 2. of the laws of motion, that when two bodies are moved with equal ve-

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locities,

locities, the moving forces must be proportional to the quantities of matter which they contain. But it appears from the preceding lemma, that this is the case of bodies moved by the force of Gravity. Therefore the moving forces of Gravity are proportional to the quantities of matter moved.

Schol. 1. Hence the ratio of the quantities of matter in two bodies is discoverable by a comparison of their gravities or weights*.

Schol. 2. † No variation of the accelerating force of Gravity, that is, of its force considered in regard only to the velocities which it generates, can be discovered in any of those distances from the surface, to which we can project bodies, and it is therefore considered as a constant force in regard to all those velocities which it generates in the neighbourhood of the earth. We may accordingly apply to the Gravity of bodies near the surface of the earth whatever properties may be proved to belong to the operation of a constant force. To discover these properties it is convenient to use the instrument invented by Mr. Atwood, in which

* This is not accurately true, except of bodies near those parts of the earth's surface, which are at equal distances from the Equator: for it will be shewn, that in different latitudes the force of Gravity generates different velocities. The only force, which is invariably proportional to the quantity of matter, is the *Vis Inertiae*.

† The accurate observations of Messrs. Bouguer and De la Condamine have indeed ascertained that on the summits of the highest mountains the accelerating force of Gravity is sensibly diminished. Their observations shall be described hereafter, and compared with the reasoning of Sir I. Newton.

the

the weight of the body moved is counterbalanced, and a small moving force may be employed. "The most obvious method," as this ingenious writer * remarks, "would be to observe the actual descent of a heavy body, as it falls towards the earth by its natural Gravity: but in this case it is manifest, that on account of the great velocity generated in a few seconds of time, the height from which the [observed body falls must be considerable." Dr. Defaguliers, who in this manner observed the descent of a leaden ball from the inner cupola of St. Pauls Church, the altitude of which is 272 feet, found that the air's resistance greatly obstructed the accuracy of experiments on bodies descending through such considerable altitudes; and that, if the altitudes were diminished, the times of motion would be so small, as to render the observation of them very precarious and unsatisfactory. Beside this, "there are no means of separating the mass moved from the moving force. We cannot therefore apply different forces to move the same quantity of matter or the same force to different quantities of matter. Moreover, the accelerating force being constant and inseparable from the body moved, its velocity will be continually accelerated, so as to render the observation of the velocity acquired at any given instant impossible." All these inconveniences are obviated by the instrument invented by Mr. Atwood, and represented in fig. 1. plate 1. By this we are enabled to establish the following properties, as belonging to constant

* Treatise on the Rectilinear Motion and Rotation of Bodies, Sect. 7.

forces, and consequently to Gravity near the surface of the earth.

1. If the same constant force act upon the same quantity of matter, the spaces described from rest are in a duplicate ratio of the times of the motion.

In Mr. Atwoods experiment the box *A* with the included weights weighed 27 quarters of an ounce weight, and the other 26. The moving force was therefore a constant one of 1 quarter of an ounce weight, and the quantity of matter moved invariable.

The square stage was then successively fixed at the distances of * 3, 12, and 27 inches from the top of the scale; and the box *A*, being let fall from that point, arrived at the stage in 1, 2 and 3 seconds. But $3 : 12 :: 1 : 4$; and $3 : 27 :: 1 : 9$.

2. If different moving forces act on the same quantity of matter for the same time, the spaces described will be proportional to the moving forces†.

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* Since, in computing the quantity of matter moved, allowance must be made, as has been already observed, for the inertia of the wheels; the numbers, in this and the following experiments, are only adapted to such instruments, as shall have wheels of the same degree of inertia. However, the relations of the resulting numbers will be the same in any instrument.

† Hence it follows, that, notwithstanding the inertness of matter, any force, however small, may move any quantity of matter however great. The inertness of a body does not hinder it from receiving motion.

This has been already established in the confirmation of the second Law of Motion.

3. During the motion of any body, moved by any constant force, a velocity is acquired, which, if continued uniformly, will cause the body to describe in an equal time a space double of that, through which the body was accelerated whilst it was acquiring that velocity.

The boxes were counterbalanced, each weighing 26 quarters of an ounce, and the box *A* was laden with an additional oblong weight of 1 quarter of an ounce. The ring was then screwed to the instrument, so as that it might intercept this additional weight at the distance of 12 inches from the top of the scale. The box *A*, laden with this weight, descended through these 12 inches in 2 seconds with a constant acceleration; and, when the additional weight had been intercepted by the ring, descended in the 2 following seconds 24 inches with a motion, which is uniform by the first Law of Motion.

4. If any constant force act on the same quantity of matter during different times, the velocities acquired will be proportional to those times.

The construction of the preceding experiment remaining, the ring was first set at the distance of 3 inches from the top of the scale, but only operates in determining the degree of velocity acquired, and in causing an equal and opposite change in the state of that body, from which it receives the motion.

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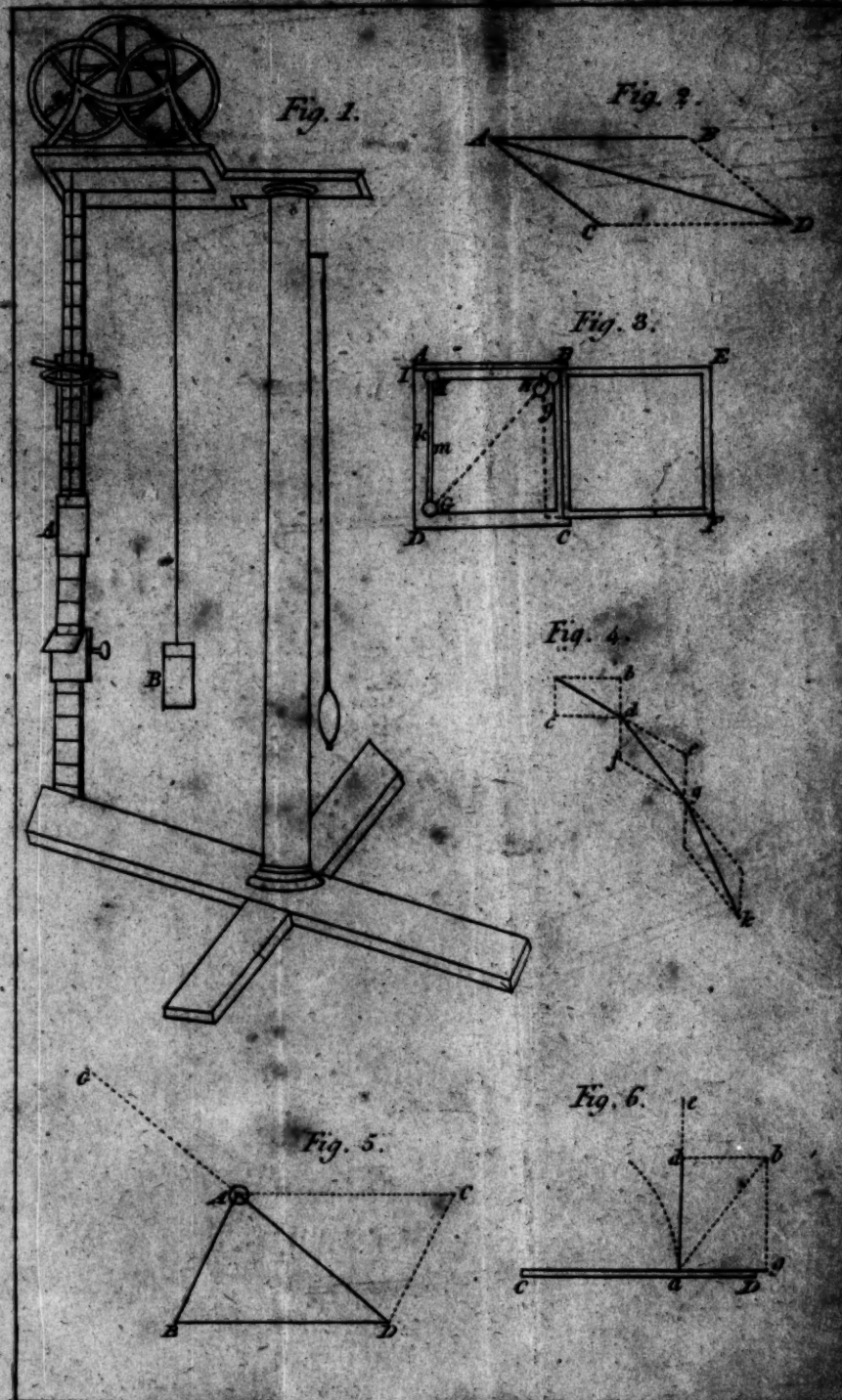
the top of the scale, and the box *A*, laden with the oblong weight, was accelerated through these 3 inches in 1 second, and moved uniformly through the following 6 inches also in 1 second. The ring was then set at the distance of 12 inches from the top, and the box *A* was accelerated through these 12 inches in 2 seconds, and afterwards moved through 24 inches also in 2 seconds, with an uniform velocity of 12 inches in 1 second. Lastly, the ring was set at the distance of 27 inches, and the box *A* was accelerated through these 27 inches in 3 seconds, and moved uniformly through the remaining 18 inches in 1 second. In these instances the times were as 1, 2, 3; and the velocities as 6, 12, 18, or in the same ratios respectively.

Hence a body moved by a constant force is uniformly accelerated, or receives equal increments of velocity in any equal times.

5. If any constant force impel the same quantity of matter through different spaces, these spaces will be in a duplicate ratio of the velocities generated.

For in the last experiment the velocities were as 6, 12, 18, or as 1, 2, 3; but the spaces were 3, 12, 27, or as 1, 4, 9.

The properties of motion accelerated by a constant force are applied to motion retarded by a constant force, as in the case of a body thrown perpendicularly upwards and resisted by its weight, by changing the velocity acquired



quired by the acceleration into the velocity destroyed by the retardation.

Hence it follows, that if a body be thrown perpendicularly upwards, it will ascend to the same height, from which it must descend in order to acquire a velocity equal to that with which it had been thrown upwards: and that the time of the ascent will be equal to the time of the descent. Of the two equal velocities, the one is generated, and the other destroyed by the constant action of the same force, the weight of the body; and consequently the body must in both cases move through equal spaces, and during equal times.

Experiments for ascertaining the properties of motion retarded by a constant force, may easily be made with the same instrument, which has been employed for proving the properties of accelerated motion. For this purpose, the box *A* with the included weights must be lighter than the box *B*, but be laden on the outside with oblong weights which shall cause it to preponderate. When the box *A* with these additional weights shall have descended to the ring, it will have acquired a certain velocity; but, the additional weights being then removed by the ring, the inferiority of the remaining weight of the box *A* will cause its motion to be continually retarded, and at length destroyed. The retarding force in these experiments is the excess of the weight of the box *B* above that of the box *A* when the additional oblong weights have been removed. These are used only for the purpose of giving the box *A* initial velocities, which are afterwards to be retarded.

6. If a body be projected in a direction oblique to that in which a constant force acts, it being supposed that the constant force acts always in parallel directions, and the resistance of the air not being considered, the body will in its course describe a regular curve.

In plate 2. fig. 1, 2 or 3, let AF be the direction in which the body is projected, and let AE represent the space through which it would in any given time be moved by the force of the projection, if no other force acted on it. Let then AB represent the space through which it would in the same time be moved by the constant force, if it had no other motion. Since then both these forces act together on the body, it will, by the 2^d cor. of the 2^d law of motion, describe a curve line passing through the point C . In the same manner, if the time be assumed greater or less, and a parallelogram under the lines representing the forces operating together during the assumed time be completed, any other point of the course of the body, as c , may be discovered. Since then the motion in the direction AF is uniform, by the 1st law of motion, Ae and AE are in the ratio of the times: but, by the 1st property of motion caused by a constant force, Ab and AB are in the duplicate ratio of the same times: consequently Ab and AB are in the duplicate ratio of Ae and AE , or of bc , and BC . Since then the several parallel lines bc , BC , &c. through whose extremities the curve passes, have a certain determinate relation to their distances from the point A in the curve, viz. to Ab , AB , &c. the course of the body is a regular curve.

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The curve, which would be described in this case, is called a parabola.

This theory is very remote from the truth in regard to all bodies projected with very great velocities, such as those which are communicated by gunpowder to bullets. In these cases the *resistance of the air will cause the projected bodies to describe paths very different from parabolas.

* The resistance which the air gives to bodies moving with very great velocities, arises from several causes. The moving body must in all cases, whether the velocity be great or small, communicate motion to the air, and by the reaction of the air must suffer a corresponding retardation. But, when the velocity of the moving body is very great, other circumstances very considerably increase the resistance.

1. A bullet, discharged from a musket or cannon, on account of its great velocity leaves behind it, during its passage through the air, either a partial or an absolute vacuum; for the air will not by its elasticity move with sufficient celerity to fill up the space deserted by the ball in its passage. The ball therefore is resisted by the whole pressure of the incumbent atmosphere, which is, from the nature of fluidity, propagated in every direction; and is not sustained by a similar pressure from behind. This force can never exceed the weight of the atmosphere.

2. The air, being condensed before the ball, exerts a force of elasticity against it in proportion to the compression. This force is increased without any limit, in proportion to the velocity of the moving body, until at length it becomes sufficient to stop or even repel it. The latter, as Mr. Atwood remarks in his *Treatise on the Rectilinear Motion and Rotation of Bodies* sect. 5. prop. 6. "is observed frequently to happen when very small shot are discharged by a large quantity of powder; in which case the shot return back in a direction contrary to that in which they were projected."

In this theory of the effects produced by Gravity near the surface of the earth, which was invented by Galileo, it is assumed that the force is constant; and this supposition is in all such cases sufficiently exact. But it has been demonstrated by Sir I. Newton, that it is actually diminished, in receding from the center of the earth, in the inverse duplicate ratio of the distances from the center: that is, if any two right lines, or numbers, express the ratio of two distances from the center of the earth, and the squares of the right lines be found by construction, or of the numbers by multiplication, the inverse ratio of these squares will be the ratio of the forces, with which Gravity would at those distances accelerate any body towards the center of the earth. Thus at a double distance the force of Gravity would be reduced to one fourth

It is obvious that the resistance of the air, arising from these causes, must occasion a deviation from the parabola, which the body would otherwise have described; since the path was proved to be a parabola, on the supposition that the velocity of projection continued uniform.

But there is still another way, in which the resistance of the air causes an irregularity in the course of a bullet, beside diminishing its velocity of projection. The bullet, by friction against the sides of the barrel from which it is discharged, acquires a rotatory motion round some one of its own diameters: and as this rotatory motion will on the one side of the bullet conspire with its progressive motion, the resistance of the air will on that side be increased; whilst on the other side, as the rotatory motion withdraws the surface of the bullet from the opposing air, the resistance will be diminished. Hence arises an inequality of the air's resistance, which must necessarily cause the bullet to deviate from its course. If the axis of the rotation be perpendicular to the horizon, the deviation will be made to the right or left: if the

axis

fourth, the inverse ratio of the squares of the distances being the ratio of 4 to 1, or of 1 to one fourth; at a treble distance the force would be reduced to one ninth, the inverse ratio of the squares of the distances being that of 9 to 1, or of 1 to one ninth. The proof of this principle is circuitous, because, though later observations have indeed discovered a diminution of Gravity on the summits of high mountains sufficiently corresponding with the theory, yet the distance of the earth's surface from the center is so great, being about 4,000 miles, that the difference of the distances of any two places near the surface from the center can have but a very inconsiderable ratio to the whole distances; and therefore even the inverse duplicate ratio of those distances will differ very little from a ratio of equality. When it shall be considered, that the observation of even this small diminution

axis be horizontal and perpendicular to the direction of the bullet, the deviation will be made upwards or downwards: and so in other cases.

To prevent this deviation *rifled barrels* have been invented. These are in their insides cut with a very open spiral, and charged with bullets which fit the barrels very closely. Whilst a bullet is forced through a rifled barrel, it acquires from the indenture a rotatory motion round an axis coincident with the line of its direction; and is consequently exempted from the irregularity which has been described. "It must be observed however" says Mr. Robins, who first conceived just ideas on this subject, "that though the bullet impelled from such a piece keeps for a time to its regular track with sufficient nicety; yet if its flight be so far extended, that its track is much incurvated, it will then often undergo considerable deflections. This, according to my experiments, arises from the angle at last made by the axis on which the bullet turns, and the direction in which it flies: for that axis continuing nearly

of Gravity is subject to the operation of causes by which it is disturbed, it will be evident that the comparison of Gravity with those forces which cause the distant movements of the heavenly bodies, will afford a much more satisfactory method of determining the law of its diminution. But another considerable advantage arises from this comparison. We learn from it that the same force of Gravity or Weight, which is the object of our ordinary observation, is extended throughout the universe, and is the active principle which maintains all the great motions of the whole system of nature. Sir I. Newton's method of demonstrating that the force of Gravity varies in the inverse duplicate ratio of the distances from the center of the earth, consists in proving 1. that the moon is, in her motion round the earth, urged by a force tending towards the earth, which varies according to this law; and 2. that this force is the same with Gravity. These positions shall be considered in the two following sections.

nearly parallel to itself, it must necessarily diverge from the line of the flight of the bullet, when that line is bent from its original direction; and when it once happens that the bullet whirls on an axis, which no longer coincides with the line of its flight, then the unequal resistance formerly described will take place, and the deflecting power hence arising will perpetually increase as the track of the bullet, by having its range extended, becomes more and more incurvated." To remedy this he proposes that bullets should be made of an oval form, so that, their heavier ends going foremost, their longer axes may be kept in the lines of their flight; in the same manner as an arrow constantly lies in the line of its direction. Robins's Mathematical Tracts, vol. 1. p. 337.

Sect. 2. *Of the centripetal force of the Moon.*

That some force is continually necessary for retaining a planet in its orbit, is evident from the first law of motion. Each planet, whatever impulse might have been given to it, must, agreeably to that law, move uniformly in a right line, unless some force be continually exerted to draw it from the direction, in which it is moving at each instant of time. The moon therefore, which revolves round the earth, must accordingly be continually influenced by some force drawing it from a rectilinear course, and compelling it to circulate in an orbit. The first enquiry concerning this force relates to its direction; for ascertaining which the following proposition must be premised.

Prop. 1. *Every body, which, in its revolution round another, describes equal areas in equal times, is urged by a force tending towards that other body, which is therefore called a centripetal force.*

Let the time of the revolution be conceived to be divided into equal parts indefinitely small, so that the force may be imagined to make one impression on the moving body in each of these instants of time; and in the first instant, let the body be supposed to move from A to B , plate 2. fig. 4. describing in regard to the body S the area ASB . In the next instant the body would, if no force acted on it, move in the same direction to c , describing Bc equal to AB , agreeably to the first law of motion; in which case the area BSc described in the second instant would

would be equal to the area ASB described in the first (Eucl. book 1. pr. 38).

But at B let the force act with an impulse, which shall cause the body to deviate from the direction AB into some other, as BC , yet so as that the area BSC , which it will describe in regard to the body S in the second instant, may be equal to ASB described in the first. Since then the area BSC is equal to ASB , it must be equal to BSc . Therefore (Eucl. book 1. pr. 39) the right line cC , which is parallel to the direction in which the force was exerted upon the body at B , must be parallel to BS . Consequently, that force was there exerted in the direction BS . In the same manner it may be shewn, that at any other points C, D , &c. at which the body arrives in the several successive instants, it is urged by forces acting in the directions CS, DS , &c. If now the force be conceived to act continually, the perimeter $ABCD$ will be converted into a curve line, and the force, continually changing its direction, will always tend towards the body S .

Cor. The moon is retained in the orbit which she describes round the earth, by a force directed towards the earth.

By comparing the apparent diameters of the moon in different parts of her orbit, the ratio of her distances from the earth, which is the reciprocal ratio of the apparent diameters, is discovered. But the angular motion of the moon round the earth may also be observed; and

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* by comparing the ratio of the angles described by her round the earth in equal times, with the ratio of her distances, it is proved that she describes round the earth equal areas in equal times. Therefore, by the preceding propo-

* The areas must be equal, if it be observed, that the angles described in equal times are reciprocally in the duplicate ratio of the distances.

To prove this it must be premised, that a ratio compounded of a direct and of an inverse ratio, may be expressed by that of two fractions, because it appears from Arithmetick, that two fractions have to each other the ratio compounded of the direct ratio of the numerators and of the inverse ratio of the denominators.

Let A, a express two arches described by the moon in equal times, and D, d her distances at those times. The angles, which she in the same times describes, must be in the ratio compounded of the direct ratio of the arches and of the inverse ratio of the distances; that is,

$:: \frac{A}{D} : \frac{a}{d}$. If then it be observed, that they are in the reciprocal

duplicate ratio of the distances, or $:: \frac{1}{D^2} : \frac{1}{d^2}$; the former

ratio must be the same with the latter, or $\frac{A}{D} : \frac{a}{d} :: \frac{1}{D^2} : \frac{1}{d^2}$.

Therefore, by alternating $\frac{A}{D} : \frac{1}{D^2} :: \frac{a}{d} : \frac{1}{d^2}$; and, by mul-

tiplying the terms of each ratio by a common multiplier, D or d ,

$A : \frac{1}{D} :: a : \frac{1}{d}$; and, by alternating, $A : a :: \frac{1}{D} : \frac{1}{d}$.

or the arches are in the reciprocal ratio of the distances. But in this case the areas must be equal. For if the equal times be assumed indefinitely small, the areas described in those times may be considered as right-lined triangles, whose altitudes are the distances, and bases are the arches. Since therefore these are reciprocal, the triangles must be equal (Eucl. lib. 6. pr. 15.) Hence it follows, that the areas described in any longer equal times must be equal, since they may be considered as the sums of equal numbers of those indefinitely small areas, which have been proved equal.

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sition, the moon is urged by a force tending towards the earth.

It having been proved, that the moon is retained in her orbit by a force constantly directed towards the earth, it

must now be shewn that this force, at different distances from the center of the earth, varies in the reciprocal duplicate ratio of those distances. Sir I. Newton has demonstrated this from a consideration of the elliptic form

of the moons orbit. He has * proved that since the moon, revolving in an elliptic orbit round the earth placed in one of the foci, is urged by a force directed towards that focus, this force must vary in the reciprocal duplicate ratio of the distances from it. The same thing he has proved more simply, and yet in a manner sufficiently satis-

factory, that the moons deviation from a regularly elliptic orbit must be ascribed to the perturbing force of the sun. If this force were exerted equally, and in parallel directions, upon the earth and moon, it could have have no effect in regard to their relative positions. The form of the moons orbit therefore is not affected by the mere circumstance of her moving with the earth round the sun in the space of a year. The suns force is only so far a perturbing force, as it is exerted unequally, or in directions not parallel or coincident. The former happens whenever, in the moons revolution round the earth, the distances of those bodies from the sun are unequal; the nearer body being more powerfully attracted: the latter happens in every point of the orbit except the syzygies; and in those also, except when the line of the nodes is coincident with the syzygial line.

* Philosophiæ Naturalis Princ. Mathem. lib. 3. pr. 3. It is there observed, that the moons deviation from a regularly elliptic orbit must be ascribed to the perturbing force of the sun. If this force were exerted equally, and in parallel directions, upon the earth and moon, it could have have no effect in regard to their relative positions. The form of the moons orbit therefore is not affected by the mere circumstance of her moving with the earth round the sun in the space of a year. The suns force is only so far a perturbing force, as it is exerted unequally, or in directions not parallel or coincident. The former happens whenever, in the moons revolution round the earth, the distances of those bodies from the sun are unequal; the nearer body being more powerfully attracted: the latter happens in every point of the orbit except the syzygies; and in those also, except when the line of the nodes is coincident with the syzygial line.

factory,

factory,* by arguing analogically from the law of the centripetal forces of several bodies revolving round a common central body; as of the primary planets which revolve round the sun, or of the satellites which revolve round the same primary planet. In these cases we are enabled to discover the variations of the force tending to the common central body, by comparing the motions of the bodies which at different distances revolve round it; and, although the moon is the only body which describes an orbit round the earth, it may reasonably be inferred from the similarity of her motion to that of each of the bodies revolving round a common central body, that the force by which she is retained in her orbit, is of the same kind with those which urge them towards their common center, and would, if the earth were attended by more than one satellite, be found to observe the same law of variation. For the estimation of these forces it is not necessary to attend to the elliptic forms of the orbits of the planets, but they may be considered as revolving in circles, whose radii are the mean distances. Nor is it necessary to consider the masses of matter contained in the moving bodies; because it has been already shewn (Law 1.) that Gravity, at the same distance from the center of the earth, acts in proportion to the quantities of matter which bodies contain, and the inquiry is only made concerning the variation of its accelerating force,

* This was the manner, in which Sir I. Newton himself first discovered the variation of Gravity, though he afterwards chose rather to deduce it from the form of the lunar orbit, as a more direct method. *Encyclopædia*, Art. Newton.

that is, of its force considered only in regard to the velocities which it generates, and not in regard to the momenta of the moving bodies.

Prop. 2. The accelerating forces, by which bodies are compelled to describe circular orbits with uniform motions, tend towards the centers of the orbits, and are proportional to the versed sines of the arches which the bodies describe in equal times indefinitely small.

These forces tend to the centers of the circles (prop. 1.) because, on account of their uniform motions, the bodies describe respectively round them equal areas in equal times. The forces therefore at *A* and *a* (plate 2. fig. 5. and 6) act in the directions *AC*, *ac*. Let *AB*, *ab* represent arches described in equal times indefinitely small, so that the bodies may be supposed to receive each a single impulse from its centripetal force during those times. Since then the bodies would, if no centripetal forces acted on them, move in the directions of the tangents agreeably to the first law of motion, if *BD*, *bd* be drawn parallel to the tangents, *AD*, *ad* will be the spaces through which the bodies are impelled by their centripetal forces in equal times, and are consequently proportional to those forces.

The equal times must be conceived indefinitely small, so as to admit but single impulses of the forces; partly because the forces tending towards the centers of the orbits are perpetually changing their directions, and partly
because

Because they are to be estimated only as they act in the peripheries of the circles.

Prop. 3 The accelerating forces are in the ratio compounded of the duplicate ratio of the arches described in any equal times directly, and of the simple ratio of the radii reciprocally.

By the preceding, the forces are in the ratio of the versed sines AD , ad . But since the chord AB is (Eucl. book 6. pr. 8. cor.) a mean proportional between AE and AD , $ABq = AE \times AD$ (Eucl. book 6. pr. 17), and therefore $\frac{ABq}{AE} = AD$. In the like manner $\frac{abq}{ae} =$

ad . Consequently, the forces are also in the ratio of $\frac{ABq}{AE}$, $\frac{abq}{ae}$. But the radii AC , ac have the same ratio as the diameters AE , ae , and may therefore be substituted for them without altering the ratio of the fractions.

Therefore the forces are in the ratio of $\frac{ABq}{AC}$ to $\frac{abq}{ac}$.

And, since AB , ab are the chords of arches supposed to be described in times indefinitely small, they may be considered as coincident with the arches. Since then it appears from Arithmetic, that two fractional quantities are in the ratio compounded of the direct ratio of the numerators and of the reciprocal ratio of the denominators, the forces, which have been shewn to be proportional to the fractional quantities $\frac{ABq}{AC}$, $\frac{abq}{ac}$, are in the

ratio compounded of the direct ratio of ABq to abq , or of

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Because they are to be estimated only as they act in the peripheries of the circles.

Prop. 3 The accelerating forces are in the ratio compounded of the duplicate ratio of the arches described in any equal times directly, and of the simple ratio of the radii reciprocally.

By the preceding, the forces are in the ratio of the versed sines AD , ad . But since the chord AB is (Eucl. book 6. pr. 8. cor.) a mean proportional between AE and AD , $ABq = AE \times AD$ (Eucl. book 6. pr. 17), and therefore $\frac{ABq}{AE} = AD$. In the like manner $\frac{abq}{ae} =$

ad . Consequently, the forces are also in the ratio of $\frac{ABq}{AE}, \frac{abq}{ae}$. But the radii AC , ac have the same ratio as the diameters AE , ae , and may therefore be substituted for them without altering the ratio of the fractions.

Therefore the forces are in the ratio of $\frac{ABq}{AC}$ to $\frac{abq}{ac}$.

And, since AB , ab are the chords of arches supposed to be described in times indefinitely small, they may be considered as coincident with the arches. Since then it appears from Arithmetic, that two fractional quantities are in the ratio compounded of the direct ratio of the numerators and of the reciprocal ratio of the denominators, the forces, which have been shewn to be proportional to the fractional quantities $\frac{ABq}{AC}, \frac{abq}{ac}$, are in the ratio compounded of the direct ratio of ABq to abq , or of

the direct duplicate ratio of the arches described in equal times indefinitely small, and of the reciprocal ratio of the radii AC , ac . But, on account of the uniformity of the velocity of each body, the arches described in these equal times are proportional to the arches described in any other equal times. Therefore the forces are in the ratio compounded of the duplicate ratio of the arches described in any equal times directly, and of the simple ratio of the radii reciprocally.

Cor. The forces are in the ratio compounded of the duplicate ratio of the velocities directly, and of the simple ratio of the radii reciprocally; that is, if V , v represent the velocities; and R , r the radii; as $\frac{V^2}{R}$ to $\frac{v^2}{r}$.

Since the arches described in equal times are, on account of the uniformity of the velocity of each body, proportional to the velocities (see page 18); the forces, which have been proved to be in the ratio compounded of the duplicate ratio of the arches described in equal times directly, and of the simple ratio of the radii reciprocally, must also be in the ratio compounded of the duplicate ratio of the velocities directly, and of the simple ratio of the radii reciprocally.

Prop. 4. If the squares of numbers expressing the ratio of the periodical times of two bodies revolving in circular orbits, be proportional to the cubes of numbers expressing the ratio

ratio of their distances from the centers, their centripetal forces are in the reciprocal duplicate ratio of those distances.

Since the velocities are uniform, they are in the ratio compounded of the ratio of the spaces described, or peripheries, directly, and of the times of describing them, or periodical times, reciprocally: for uniform velocities are greater in the same proportion in which the spaces described are greater, or the times of motion less. Therefore, since a ratio compounded of a direct and of a reciprocal ratio may be expressed by the ratio of two fractional quantities, on account of the property of such quantities mentioned in the preceding proposition, if the peripheries be expressed by P, p , and the periodical times by T, t ;

$V : v :: \frac{P}{T} : \frac{p}{t}$; or, putting the ratio of the radii for that of the peripheries (Theor. fel. ex Arch. prop. 7. Tacq. Eucl.) $:: \frac{R}{T} : \frac{r}{t}$; and consequently

$Vq : vq :: \frac{Rq}{Tq} : \frac{rq}{tq}$. Since therefore the forces are in general (by the cor. of the preceding prop.)

$:: \frac{Vq}{R} : \frac{vq}{r}$, by substituting for the ratio of

Vq to vq that of $\frac{Rq}{Tq}$ to $\frac{rq}{tq}$, $V : v :: \frac{Rq}{Tq \times R} :$

$\frac{rq}{tq \times r}$, or, by simplifying the fractional quantities

$:: \frac{R}{Tq} : \frac{r}{tq}$. But, by the hypothesis, the ratio of the squares of the times is in this case the same

with

with that of the cubes of the distances, or the ratio of Tq to tq the same with that of R^c to rc . If then this ratio be substituted for the former, as being equivalent to it, the forces, which have been proved to be in the ratio of

$\frac{R}{Tq}$ to $\frac{r}{tq}$, must also be in the ratio of $\frac{R}{R^c}$ to $\frac{r}{rc}$; or, by again simplifying the terms, in the ratio

of $\frac{1}{Rq}$ to $\frac{1}{rq}$, that is, in the reciprocal duplicate ratio of the distances.

Cor. The centripetal force of the moon varies in the reciprocal duplicate ratio of the distances from the earth.

It has been ascertained by astronomical observations, that the periodical times and distances of the primary planets observe the law supposed in this proposition; as also the periodical times and distances of the satellites revolving round the same primary, as of those of * Jupiter, or Saturn. Therefore, by the proposition, the centripetal forces of the several planetary bodies, which revolve round a common center, are in the reciprocal duplicate ratio of the distances. Since therefore the revolution of the moon round the earth is a phenomenon of the same kind with the revolutions of the primary planets round the sun, and of the satellites of Jupiter or Saturn round their respec-

* The phenomena of the satellites of the Georgium Sidus have not yet been observed with sufficient accuracy, to enable us to deduce inferences from them.

five primaries; it may be inferred, that the centripetal force of the moon must observe the same law in regard to different distances from the earth.

Prop. 5. *To determine what would be the effect of the moon's centripetal force, if it were exerted near the surface of the earth.*

The radius of the moon's orbit is * assumed to be equal to 60 semidiameters of the earth; and, since the number of miles contained in a semidiameter of the earth is ascertained by the measurement of a degree of a meridian, the number of miles contained in the radius of the moon's orbit is also known. Hence is known the number of miles contained in the periphery of the moon's orbit, the ratio of the radius of a circle to its periphery being known with sufficient accuracy (Theor. sel. ex Arch. prop. 6. Tacq. Eucl.); and consequently, since the moon's periodical time is known by observation, the space described by the moon in one minute of time is likewise known. But since this arch is known, its versed sine may also be discovered; and this versed sine is the space through which the moon is impelled by her centripetal force in a

* The true distance exceeds this by one half of a semidiameter of the earth; but the lesser distance is assumed, because, in the application of the preceding propositions, no allowance is made for that motion, with which the earth itself revolves round the common center of Gravity of the earth and moon. The difference between the true and assumed distance is just such as corresponds to the inaccuracy of the supposition that the earth has no motion round that center. Vide Newtoni Princ. lib. 3. pr. 4.

minute of time. This space is, by this computation, found to be equal to 15 Parisian feet 1 inch 1 line $\frac{4}{9}$. Since then the moon's centripetal force in different distances from the center of the earth varies, by the preceding corollary, in the reciprocal duplicate ratio of those distances, it must at the surface of the earth, or at the distance of 1 semidiameter of the earth from its center, be increased 60×60 times, this being the reciprocal duplicate ratio of the distances 1 and 60. Wherefore a body near the surface of the earth, urged by the moon's centripetal force thus augmented, would in a minute of time be moved through 60×60 times 15 Parisian feet 1 inch 1 line $\frac{4}{9}$. But this space is to the space, through which it would by the same force be moved in a second of time in the duplicate ratio of the times, by the 1st property of motion accelerated by a constant force, or as 60×60 to 1. Therefore a body near the surface of the earth, urged by the moon's centripetal force, would in a second of time be moved through 15 Parisian feet 1 inch 1 line $\frac{4}{9}$.

To

• It is obvious that, to cause a body to move in a given orbit with a given centripetal force, some determinate velocity of projection is necessary. If the velocity were less, the centripetal force would prevail, and the body would approach nearer to the center; and if it were greater, it would prevail, and the body would recede to a greater distance from the center. In a circular orbit the requisite velocity is equal to that, which the body would acquire, if it were urged through a space equal to half of the radius by a constant force equal to the given centripetal force. For let *AL* (plate 2. fig. 5.) be the space, through which the body must be urged by the constant force, that it might acquire a velocity equal to that with which it would revolve in the

To prove that the centripetal force of the moon is the same with Gravity, it is now necessary that the intensity of the latter at the surface of the earth should also be ascertained: that is, that it should be determined, through what space a body falling freely would descend in a second of time. If this should be found to be the same as that already ascertained for the lunar force, the proof will be complete. On account of the reasons mentioned in page 27, the actual operation of Gravity is most accurately ascertained by the assistance of a Pendulum, the properties of which must therefore be considered.

SECT. 3. Of the Pendulum.

Prop. 1. *The force, with which a body by its Gravity descends along an inclined plane, is to the whole force of its*

the circle; and let AF be the arch described with that velocity in the same time. By the 3rd property of motions generated by constant forces, $AF = 2 AL$. But, by the 1st property, AL is to AD in the duplicate ratio of the times; and therefore, since the arches AF , AD , described uniformly in the same times respectively, are proportional to these times, $AL : AD :: AF^2 : AB^2$. Consequently $AL \times AE : AD \times AE :: AF^2 : AB^2$; and, by alternating, $AL \times AE : AF^2 :: AD \times AE : AB^2$. But $AD \times AE = AB^2$, the indefinitely small arch coinciding with its chord. Therefore $AL \times AE = AF^2$. And since it was before shewn, that $AF = 2 AL$, $AF^2 = 4 AL^2$. Therefore $AL \times AE = 4 AL^2$; and consequently, both quantities being divided by AL , $AE = 4 AL$, and $AC = 2 AL$.

*Gravity, with which it falls perpendicularly, as the * height of the plane to its length.*

Let AC (plate 2. fig. 7.) be the plane, BC its base parallel to the horizon, AB its perpendicular height, and D the place of the body. From the point D draw DE parallel to AB and let it represent the whole Gravity of the body. Since the body is partly supported by the plane, its Gravity can only operate by causing it to descend along the plane; that is, it can only act in a direction parallel to the plane. To determine, therefore, what part of the whole Gravity of the body is exerted in causing its descent, this force must, agreeably to, cor. 4 of the 2nd law of motion, be resolved into two, one of which shall be parallel to the plane, and the other perpendicular to the former. For this purpose, from E draw EF perpendicular to the plane, and the whole force represented by DE will be equivalent to the combined action of two forces represented by DF , FE . Of these DF alone can operate in causing the descent of the body, the other being counteracted by the plane. But the triangle DEF is similar to the triangle ABC . Therefore, since the force of the descent along the plane is to the whole force of Gravity as DF to DE , it must also be to it as AB to AC .

Cor. The force, with which a body by its Gravity descends along the same inclined plane, is constant.

* That is, if the length of the plane be considered as radius, the force of the descent along the plane is to the whole force of Gravity, as the sine of the angle of elevation to radius.

In

In every part of the plane it has, by the prop. to the whole force of Gravity the same ratio, namely that of the height of the plane to its length. Since, therefore, the whole force of Gravity may be considered as constant (see page 26), the force of descent along a plane, whose inclination is unvaried, must be also considered as constant.

Schol. Hence, those properties of the motion produced by a constant force, which have been proved in the 1st section, may be applied to the motion of a body descending by its Gravity along the same inclined plane.

Prop. 2. * *The space described in any given time by a body descending along an inclined plane, is to the space through which it would fall perpendicularly in the same time, as the height of the plane to its length.*

* As the following propositions treat only of the various effects produced by Gravity, which (lemma 1. sect. 1.) accelerates all bodies equally; it is not necessary to consider the quantities of matter moved, and the forces are accordingly compared only as accelerating forces. No allowance is made in these propositions for the friction of the surfaces, on which the bodies move: nor is such an allowance necessary when the resulting conclusions are only applied to the case of the Pendulum, in which the surface is imaginary. The resistance of the air is also neglected; but this, though it tends gradually to destroy the motion of the Pendulum, does not sensibly alter the time of its vibration through a very small arch, the resistance in this case being very small. In the following propositions the motions considered are supposed to begin from a state of rest.

By the 2nd property of motions produced by constant forces, the spaces described in the same time must be proportional to the forces; and these are (prop. 1.) as the height of the plane to its length.

Cor. 1. To determine the length of that part of an inclined plane, along which a body would descend in the same time, in which it would fall down the height of the plane.

In fig. 8. plate 2. from *B* draw *BD* perpendicular to the plane, and *AD* will be the part required. For (Eucl. book 6. prop. 8, cor.) $AD : AB :: AB : AC$; that is, *AD* is to the height of the plane in the ratio determined by this proposition for the spaces described in equal times.

Cor. 2. If a circle be perpendicular to the horizon, and chords be drawn in it from the lower extremity of its vertical diameter, the times of descending along them, considered as inclined planes, would be equal.

Let *AHFB* (fig. 1. plate 3) be the circle, and *AB* its vertical diameter. Let any chords, as *BH*, *BF*, be drawn in it from *B*; and from *A* draw other chords *Ab*, *Af*, respectively parallel to the former, and, when produced, meeting in *G* and *C* the horizontal line *BGC*. If then the lines *Bb*, *Bf* be drawn, because (Eucl. book 3. pr. 31.) the angles in a semicircle are right, *Ab* would be, by the preceding corollary, the space described by a body descending along *AbG*, in the same time in which it would fall down *AB*; and *Af* in like manner would

would be described in the time of the same fall. Therefore the chords Ab , Af would be described in equal times. But these chords are respectively parallel, and therefore also respectively equal, to the given chords BH , BF . Since they are respectively parallel, the accelerating forces will be respectively equal, and will therefore, by the 2^d property of motions produced by constant forces, cause bodies to move along the equal chords in times also respectively equal. The given chords BH , BF will therefore be described in equal times; and also in the same time, in which a body would fall down the diameter AB .

Prop. 3. *The time in which a body descends along an inclined plane, is to the time in which it would fall down the height of the plane, as the length to the height, or as the spaces described.*

For (fig. 8. plate 2.) the time of the descent from A to C is to the time of the descent from A to D , in the subduplicate ratio of AC to AD , by the 1st property of motions produced by constant forces. But, by the 1st cor. of the preceding proposition, the time of the descent from A to D is equal to the time of the fall down the height AB . Therefore the time of the descent from A to C is to the time of the fall from A to B , in the subduplicate ratio of AC to AD ; that is, because (Eucl. book 6. prop. 8. cor.) AB is a mean proportional between AC and AD , in the ratio of AC to AB .

Lemma.

Lemma. The velocities generated by constant forces are in the ratio compounded of the ratio of the forces, and of the ratio of the times during which they act.

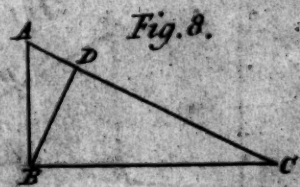
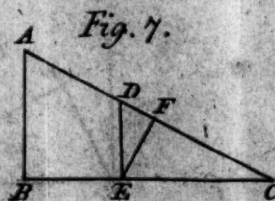
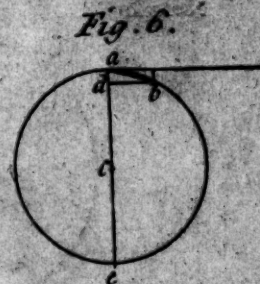
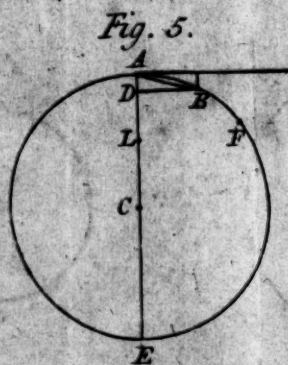
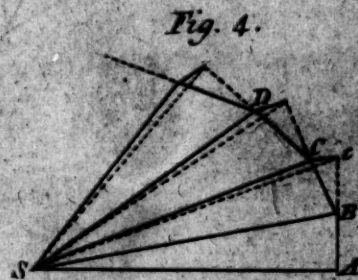
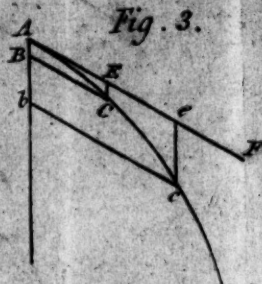
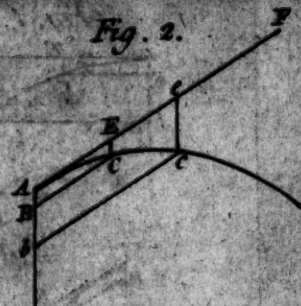
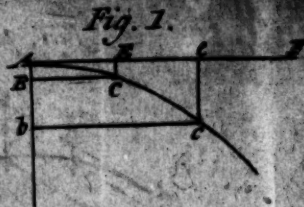
For, by the 2nd law of motion, the velocities are proportional to the forces, if the times be equal; and, by the 4th property of motions produced by constant forces, the velocities are proportional to the times, if the forces be equal. They must therefore be, when both the times and forces are different, in the ratio compounded of these ratios.

Prop. 4. The velocity acquired at the end of the descent along an inclined plane, is equal to the velocity acquired at the end of the fall down the height of the plane.

For these velocities are, by the preceding lemma, in the ratio compounded of the ratios of the forces and times. But (prop. 1.) the force of the descent is to the force of the fall, as the height to the length; and (prop. 3.) the time of the descent is to the time of the fall, as the length to the height. These ratios are consequently reciprocal, and therefore the ratio compounded of them is a ratio of equality.

Cor. The velocities acquired in descending along inclined planes are equal, when the heights of the planes are equal.

For (plate 3. fig. 2.) the velocity acquired in descending from *A* to *C*, is, by the proposition, equal to the velocity acquired in falling from *A* to *B*; and so also is the



the velocity acquired in descending from A to G . The velocities therefore acquired in the descents along the planes are equal.

Prop. 9. If a body descend along a system of inclined planes, the velocity acquired, it being supposed that the body is not retarded by shocks received in the angles, is equal to that which would be acquired in falling down the height of the system.

Let $ABCD$ (plate 3. fig. 3.) be the system of planes; draw AE , DF parallel to the horizon; produce CB , DC till they meet AE in G and E ; and draw EF perpendicular to the horizon. Then the velocity acquired in descending from A to B , is, by the cor. of the preceding proposition, equal to that which would be acquired in descending from G to B ; and since, by the supposition, no velocity is lost in the angle B , the body will begin to descend along BC with the same velocity, whether it has descended along AB or GB . Therefore the velocity acquired at C will be the same, as if the body had descended along GBC ; and consequently the same, by the cor. of the preceding proposition, as if it had descended along EC . In the like manner, no velocity being lost at C , the velocity acquired at D will be the same, as if the body had descended along ECD , and consequently, by the preceding proposition, equal to that which would be acquired in falling down EF .

* This supposition may be made, because the conclusion derived from it is only applied to the swing of the Pendulum, in which there are no angles.

Cor. If a body descend along any curve surface, the velocity acquired is equal to that, which would be acquired in falling down its height.

For, if the planes in the system *ABCD* be increased indefinitely in number, and consequently diminished indefinitely in magnitude, the system will become ultimately a curve surface.

Schol. The preceding corollary is also applicable to the case of a Pendulum, or body suspended from a point round which it swings by its Gravity. For the string or rod, being perpendicular to the curve in each point, will sustain the same part of the weight of the body, which is in the other case supported by the curve surface.

*Prop. 6. If a Pendulum be removed from its vertical position through any arch of the * curve, which it describes in its vibration, it will, when it shall have descended to that position, ascend through an equal length of a similar arch; and the time of the ascent will be equal to that of the descent.*

For, on account of the velocity acquired in its descent, it will not remain at rest in the lowest point; and, the arches being similar and similarly situated in regard to the horizon, the forces, which retard the Pendulum in

* The proposition is expressed thus generally, because a contrivance has been used for causing Pendulums to vibrate in a curve different from a circle, namely a cycloid. Vide Cotes de Cycloide.

the several points of its ascent, are respectively equal to those which accelerate it in the corresponding points of its descent.

Prop. 7. If a Pendulum vibrate in circular arches, the part of the force of Gravity, which accelerates or retards the motion of the Pendulum, varies as the sine of the distance from the lowest point.

For let the Pendulum move in a circular arch AB (plate 3. fig. 4.) whose radius is AC . From the center C , and from A the place of the body, draw CB and AE perpendicular to the horizon; and let AE be assumed to represent the whole force of Gravity. AE must therefore be of the same length in every part of the arch. Draw the tangent AF , and EF perpendicular to it. The whole force of Gravity is, by cor. 4. of the 2^d law of motion, equivalent to two forces acting at A , represented by AF , FE ; of which the latter, being perpendicular to the tangent, is counteracted by the string or rod of the Pendulum. AF therefore represents the accelerating or retarding force, which consequently must be to the whole force of Gravity, as AF to AE . If now AD be drawn perpendicular to CB , it will be the sine of the arch AB , and, on account of the similarity of the triangles (Eucl. book 6. prop. 4.) $AD : AC :: AF : AE$. Therefore the accelerating or retarding force in the point A is to the whole force of Gravity, as AD is to AC ; and, since in these ratios the consequents are invariable, whatever point of the arch be the place of the body, the antecedents, namely

the

the accelerating or retarding force and the sine *AD*, must vary each in the same ratio.

Lemma. If unequal spaces be described with forces always proportional to the spaces to be described, the times of motion will be equal.

For, since in the first instant the forces, which are proportional to the whole spaces, may be considered as constant, the spaces described in the first instant will be proportional to the forces, by the 2nd property of motions produced by constant forces, and consequently, by the supposition, to the whole spaces. Therefore, by division of proportion, the spaces remaining to be described will be proportional to the whole spaces; and they are also, by the supposition, proportional to the forces operating in the second instant, and consequently these forces are proportional to the whole spaces. Therefore it may be shewn, that in each successive instant spaces are described, by the combination of the new forces with the motions already acquired, proportional to the whole spaces; and consequently that the whole spaces will be described in equal times.

*Prop. 8. All the vibrations of a Pendulum vibrating in circular arches are performed in * equal times, if the arches be very small.*

For,

* In a cycloidal curve all the arches, whether great or small, would be described in equal times, if the Pendulum were only a mathematical point: but since in reality every particle of a Pendulum is a distinct Pendulum vibrating by its own Gravity, and at its own distance from the point of suspension, the whole must be conceived to be collected

lected

For, by the preceding proposition, the accelerating forces in the descents are in general proportional to the sines of the distances from the lowest point: and therefore, if the arches be very small, are proportional to the arcual distances from the lowest point; since the sines may in this case be considered as coincident with the arches. The forces are therefore in all the points of the arches proportional to the spaces to be described in the descents; and consequently, by the preceding lemma, the descents will be performed in equal times. But (prop. 6.) the times of the ascents are respectively equal to those of the descents. Therefore the whole times of the vibrations are equal.

Prop. 9. *If a Pendulum vibrating in a circular arch AVB (plate 3. fig. 5.) begin to descend from L, the velocity acquired at any point M varies as the sine mx of a circular arch lx, whose radius is equal to LR the sine of LV, and versed sine lm is equal to the difference of the sines LR and MS; it being supposed that the arch LVP is very small.*

lected into a point, called the center of oscillation, at which the same force being applied would cause the same motion; and unless this point continue the same in every part of the vibration, the description of the curve will be disturbed. This is the case of every curve except a circle, because the place of the center of oscillation depends on the distances from the point of suspension, and can therefore remain unvaried only when these distances are unvaried. This advantage overbalances the inaccuracy, with which this and the following propositions are proved of circular arches. For the method of finding the center of oscillation, see Mr. Atwoods Treatise on the Rectilinear Motion and Rotation of Bodies, sect. 5. prop. 5.

For the velocity acquired in the descent from L to M is equal to the velocity acquired in falling from R to S (prop. 5. cor.); and therefore, by the 5th property of motions produced by constant forces, varies in the subduplicate ratio of RS , that is, in the subduplicate ratio of $RV - SV$; and consequently in the subduplicate ratio of this difference multiplied by the diameter DV , or of $DV \times RV - DV \times SV$. But these quantities are respectively equal to the squares of the chords LV , MV (Eucl. book 6. prop. 8. cor. and prop. 17). Therefore the velocity at any point M varies in the subduplicate ratio of $LV^2 - MV^2$; and consequently, since when the arches are very small the chords and lines may be considered as coincident, in the subduplicate ratio of $LR^2 - MS^2$, or of $IV^2 - mV^2$, or of $xV^2 - mV^2$, or of mx^2 ; that is as mx .

Cor. The velocity at M : the velocity at V : : mx : Vx .

For, by the proposition, the velocity at M : the velocity at V : : mx : Vx .

Prop. 10. The time of a vibration in the arch LVP is equal to the time in which a body would describe the semiperiphery lzp with the velocity acquired at V continued uniform; it being supposed that the arch LVP is very small.

Let MN be an indefinitely small arch, in which the velocity may be considered as uniform; take mn equal to the difference of the lines MS , NT , draw the line nt , and

and suppose a body to describe the semiperiphery isp with the velocity acquired at V continued uniform. The arch st will be described with this uniform motion in the same time, in which MN is described by the Pendulum; since it will be shewn that they are proportional to the uniform velocities with which they are described, and bodies moving with uniform velocities must in equal times describe such spaces (see page 18). Let sr be drawn parallel to mn , and the triangles Vxm xrt , xt being considered as coinciding with its tangent, will be similar (Eucl. book 6. prop. 4.) because the angles at r and m are right angles, and the angle Vxr is the common complement of the angle xrt and of the angle Vxm . Therefore $Vx : mx :: xt : sr$ or mn . But mn was taken equal to the difference of the sines MS , NY , and may therefore, on account of the smallness of the arches, be considered as equal to the difference of the arches MN . Therefore $Vx : mx :: xt : MN$, or the velocities are proportional to the spaces. The same may be shewn of all other corresponding parts of the two arches. Therefore the whole time of the vibration is equal to the time of describing the semiperiphery isp with the velocity acquired at V continued uniform.

Prop. 11. *The time of a vibration in a very small circular arch is to the time in which a body would fall down half of the length of the Pendulum, as the periphery of a circle to its diameter.*

The time of describing the semiperiphery isp with the velocity acquired at V continued uniform is to the time

of

of describing the radius IV with the same velocity, as the spaces described, because the velocity is the same; or as the periphery of a circle to its diameter. But the time of describing the semiperiphery with this velocity is, by the preceding proposition, equal to the time of the vibration; and it will be shewn, that the time of describing the radius with the same velocity is equal to the time of the fall down half of the length of the Pendulum. If a body descend along the chord LV , it will acquire the same velocity as by descending along the arch $LMNV$ (prop. 5 and cor. of prop. 6), and the time of descending along the chord LV will therefore, by the 3^d property of motions produced by constant forces, be double of that in which a body would with the velocity acquired at V continued uniformly move through an equal space. And, since the time of descending along the chord LV is (prop. 2. cor. 2) equal to the time in which a body would fall down the diameter DV , the time of falling down the diameter DV is double of that in which a body would with the velocity acquired at V continued uniformly move through a space equal to the chord LV . But, since the arch LV is very small, the sine LR , and consequently the radius IV , may be considered as equal to the chord LV . Therefore the time of falling down the diameter DV is double of the time, in which a body would with the velocity acquired at V continued uniformly describe the radius IV . But, by the 1st property of motions produced by constant forces, the time of falling down the diameter DV is also double of the time of falling down one fourth part of the length of the diameter, or one half of the length of the Pendulum,

lum. Therefore the time of falling down one half of the length of the Pendulum is equal to the time, in which a body would with the velocity acquired at V continued uniformly describe the radius IV .

Prop. 12. *The times, in which Pendulums of different * lengths perform vibrations in very small arches, are in the subduplicate ratio of the lengths of the Pendulums, the force of Gravity being unvaried.*

For the time of the vibration of each Pendulum through a very small arch is to the time, in which a body would fall down half of the length of the Pendulum, as the periphery of a circle to its diameter, by the preceding; that is in a given ratio (Theor. sel. ex Arch. prop. 6. Tacq. Eucl.) The times of the vibrations are therefore proportional to the times of the falls down the halves of the lengths of the Pendulums; and consequently, by the 1st property of motions produced by constant forces, in the subduplicate ratio of the halves of the lengths, or in the subduplicate ratio of the lengths themselves.

Cor. *The numbers of vibrations, performed in the same time by Pendulums of different lengths, are reciprocally in the subduplicate ratio of the lengths of the Pendulums.*

For the numbers of vibrations performed in the same time must be in the reciprocal ratio of the times of single vibrations.

* The lengths are the distances between the points of suspension and the centers of oscillation.

Prop. 13. If the forces of Gravity be different, the lengths of Pendulums, which perform their vibrations in equal times, will be proportional to the forces of Gravity.

For the equal times of the vibrations have (prop. 11) the same ratio to the times of the falls down the halves of the lengths of the Pendulums. The times of these falls are therefore equal; and consequently, by the 2^d property of motions produced by constant forces, the forces of Gravity are proportional to the halves of the lengths, or to the lengths themselves.

Prop. 14. If a Pendulum perform each vibration in a second, the space through which a body would fall in a second is to the half of the length of the Pendulum, in the duplicate ratio of the periphery of a circle to its diameter.

The time of the vibration, or one second, is (prop. 11) to the time of the fall down the half of the length of the Pendulum, as the periphery of a circle to its diameter. Therefore, by the 1st property of motions produced by constant forces, the spaces through which bodies fall in these times, or the space through which a body falls in a second and the half of the length of the Pendulum, must be in the duplicate of this ratio.

Scholium. The time of the vibration of a Pendulum is not altered (if the resistance of the air be not considered) by varying its weight, since Gravity accelerates all bodies equally (Law 1). A Pendulum however made

of

of a denser substance, by moving with a greater momentum in consequence of its greater quantity of matter, more effectually overcomes the resistance of the air, which remains the same whilst the bulk and velocity of the Pendulum are unvaried. In another respect also the time of the vibration of a Pendulum may be affected by the nature of the substance of which it is composed. * All substances are expanded by heat, and contracted by cold. The length of a Pendulum is therefore altered by variations of temperature, and (prop. 12) the times of its vibrations must be altered in the subduplicate ratio of its lengths. Deal-wood is found to be in a very small degree affected by changes of temperature, and is consequently well adapted to be the material of the rods of Pendulums. Mr. Nicholson mentions (Introduction to Nat. Phil. vol. 1. p. 91.) that "the wood called Sapadillo is said to be still better." Various contrivances have however been devised for the intire removal of this inconvenience, by the compensation of contrary expansions or contractions. Of these the two following seem to be the most effectual.

The Gridiron Pendulum is suspended by a combination of any convenient odd number of rods, as five, seven, or nine, in the manner represented in plate 3. fig. 6.

* There are some few exceptions from this general rule. Clay is contracted when exposed to heat. This however is attributed to the expansion of the fluid which it had contained. It has also been observed that water, though it is contracted whilst its temperature descends from the boiling point, or 212 degrees, to that of 40 degrees, is yet expanded during the farther diminution of its temperature to the freezing point, or 32 degrees. Count Rumfords Exp. Essays.

The two exterior rods are fastened in the cross bar *BC*, but the three inner rods pass loosely through holes made in the same bar. The three inner rods are all fastened in the cross bar *DE*. All the rods are fastened in the lower cross bar *FG*, except the middle one which passes freely through it, and supports the flattened ball or lens *H*. In this construction the expansion of the exterior rods depresses the bar *FG*; the expansion of the two next rods on the contrary removes the bar *DE* to a greater distance from *FG*; whilst that of the middle rod increases the distance between the bar *DE* and the lens *H*. If then the two rods between the middle and the exterior rods be made of a substance, whose expansion is so much greater than that of the others in the same change of temperature, that it may, by elevating the bar *DE*, compensate the depression of the bar *FG* by the expansion of the exterior rods, and the increased distance of the weight *H* from the bar *DE*, it is evident that the interval between the weight *H* and the point of suspension *A* will remain unchanged. For this purpose the two exterior rods and the middle one are made of steel; and the two others of a composition of zinc and silver, or of brass. The adjustment of the contrary expansions is made, from observation of the motion of the Pendulum, by varying the distance of the bar *DE* from *FG*, and consequently the ratio of the length of steel to that of the other metal used in the combination.

The other contrivance, which has been invented by Mr. * Crosshwaite of Dublin, is more simple. On a bracket *D*

* Transactions of the Royal Irish Academy, for the year 1788.

(plate 3. fig. 7.) is supported a rod of steel *B*, which has liberty to move freely upwards through staples 1, 2, 3, 4. At the upper extremity of *B* is formed a gibbet *C*, from which is suspended another rod of steel *A*, forged out of the same bar, and in all respects similar to *B*. The weight is attached to the rod *A*. It is obvious, that in this Pendulum the place of the weight is not varied by the equal and contrary expansions of the two rods. Its distance from *C* is indeed altered; but this does not affect the real length of the Pendulum, because that is determined by the application of two cheeks immoveably fixed at *K*, between which passes a small spring, by which the rod *A* is suspended.

To avoid the inconvenience of friction at the point of suspension, a small flexible spring is commonly used for connecting the Pendulum with that point. The form of a lens is chosen for the weight, that the resistance of the air may be as little as possible*.

K 2

Sect. 4.

* The Pendulum, which is the most accurate measure of time, has been proposed as the best standard for correcting the disagreements and uncertainty of the measures both of extent and of weight. The received measures of both these are derived from elementary measures incapable of precise determination, and are themselves different combinations of these elementary measures according to the arbitrary usages of different countries, and even of different parts of the same country. The Pendulum however affords a standard measure, which may at all times be ascertained with considerable accuracy. In the same part of the earth a Pendulum, which shall perform each vibration in a certain time, as one second, must be of a determinate length.

Since all the observations made for this purpose are supposed to be made in the same part of the earth, the force of Gravity must be unvaried;

Sect. 4. Of the other Laws of Gravity, and its operation throughout the universe.

Lemma. The force of Gravity at the surface of the earth, in the latitude of Paris, is such, that a body would fall in a second 15 Parisian feet 1 inch $1\frac{1}{2}$ line.

For the length of a Pendulum performing each vibration in a second in the latitude of Paris is 3 Parisian feet $8\frac{1}{2}$ lines; and (sect. 3. prop. 14.) the space through which a body would fall in a second is to the half of this length in the duplicate ratio of the periphery of a circle to its diameter.

Law 2.

ried; and (prop. 13) the lengths of Pendulums performing their vibrations in the same time are proportional to the forces of Gravity. Measures of length may therefore be invariably determined by their relation to that of a Pendulum performing each vibration in a second in some certain latitude. From these may easily be obtained, not only superficial measures; but also those of capacity, if a hollow cube, whose sides are of a determinate length, be assumed as the standard. Measures also of weight may in the same manner be regulated, by assuming as a standard a solid cube of some certain substance.

This method of ascertaining measures is not however exempt from many difficulties. "In experiments to determine the length of a Pendulum vibrating through any known portion of time employed by the earth in its rotation, we have to enquire," says Mr. Nicholson, "1st. Whether that rotation be theoretically uniform; and if not, what are the quantities and periods of irregularity, perceptible in such an experiment? 2^{ly}. What are the best methods and limits of error in measuring the length of a Pendulum between its centres of oscillation and suspension? 3^{ly}. Whether the errors from temperature can be rendered insensible in this measure of time and length; and if not, what

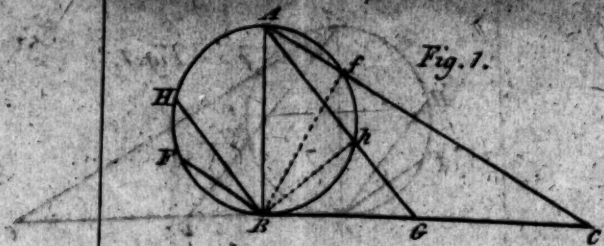


Fig. 1.

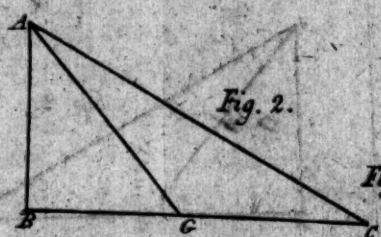


Fig. 2.

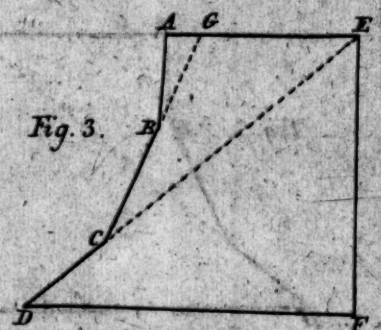


Fig. 3.

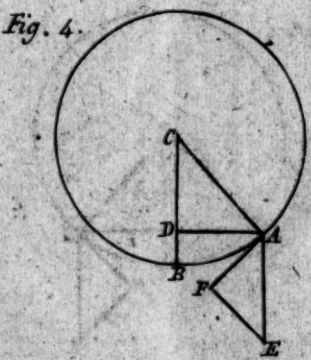


Fig. 4.

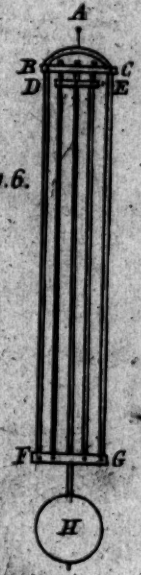


Fig. 6.

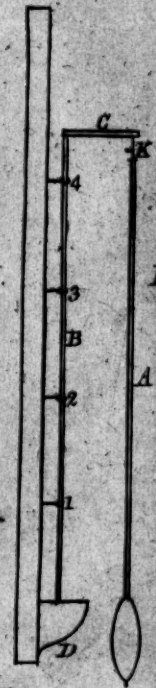


Fig. 7.

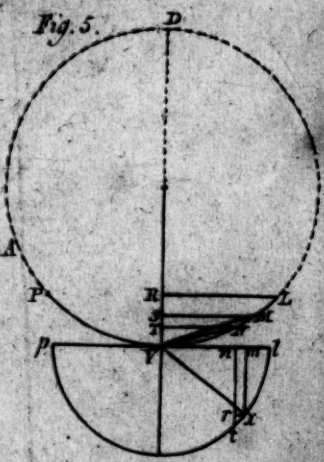


Fig. 5.

Law. 2. The accelerating force of Gravity varies in the reciprocal duplicate ratio of the distances from the center of the earth.

It has been shewn (sect 2. prop. 1. cor.) that the moon is retained in her orbit by a force directed towards the earth; and (prop. 4. cor.) that this force varies in the reciprocal duplicate ratio of the distances from the center of the earth. It may now be shewn, that this force is the force of Gravity, and consequently, that the force of Gravity varies in the reciprocal duplicate ratio of those distances. A body near the surface of the earth,

what are their limits? And above all, 4ly. What are the effects of the escapement part, or simplest maintaining power in a clock; and what are the best means of-diminishing or removing them altogether?"

Journal of Natural Philosophy, Chemistry and the Arts, for August 1797, p. 198.

The French nation have lately adopted a new system of measures, which they have chosen rather to derive from the measurement of a degree of the meridian at the mean latitude of forty five degrees. To the former method of regulating them by the Pendulum it was objected, 1st. that the division of the day into seconds is in itself arbitrary; and 2ly. that the measures thence resulting and geographical measures would have no mutual agreement, since the league could not be at the same time a multiple of the Pendulum in round numbers and an aliquot part of a degree. Mr. Nicholson however observes, that "the chief, and indeed the sole, motive of preference must consist in the greater degree of accuracy, or more perfect agreement of the results wrought out by different observers." It is certain that the measurement of lines and angles upon the earth admits of wonderful accuracy. The result of a trigonometrical computation of a space of 36574.3 feet, founded on an observation of the angles of seventeen triangles, was only about one inch short of its actual measurement.

"It

earth, urged by the moons centripetal force augmented in the reciprocal duplicate ratio of the distances, would in a second of time fall through the space of 15 Parisian feet 1 inch 1 line $\frac{4}{5}$ (sect. 2. prop. 5.) And, by the preceding lemma, a body, urged by the force of Gravity also near the surface, would fall through the space of 15 Parisian feet 1 inch 1 line $\frac{7}{9}$. The results of these two computations are sufficiently near to each other, allowance being made for the unavoidable inaccuracy of some of the principles, to warrant us in maintaining, that the two forces in the same circumstances would produce effects exactly equal and similar, and must therefore be considered as one and the same force.

This argument derived from a consideration of the moons motion has been confirmed, as nearly as circum-

"It seems probable," says Mr. Nicholson, "that there may have been some fortunate compensation of errors in this wonderful coincidence; but the result appears to prove, that the method of terrestrial admeasurement will give an original measure to be depended on for more than four places of figures, and less than five." When surfaces and solids are to be derived from linear measures, if there be any error in the original line, the error in the surface or area will be so much increased in each case, as to affect nearly the same figure of the result. The French have assumed as their fundamental unity the distance of the north pole from the equator, found by multiplying by 90 the magnitude of a degree at the mean latitude. For convenience however they have adopted also another unity, called a Metre. This measure, consisting of three feet eleven lines and forty four hundredth parts, was found after seven successive decimal divisions of the fundamental unity, of which it is therefore the ten millionth part. *Journal of Natural Philosophy, &c. ib.*

stances

stances would permit, by observations made at the greatest distance from the ordinary surface of the earth, to which we have access. Messrs. Bouguer and De la Condamine, who were sent from France to South America, in the year 1735, to measure a degree of the meridian near the equator, found that, on the summit of the mountain Pichincha, at the height of 2434 toises above the level of the sea, a Pendulum, performing each vibration in a second, was * $\frac{1}{845}$ part shorter than at the level of the sea. Since the lengths of Pendulums performing their vibrations in equal times are proportional to the forces of Gravity (sect. 3. prop. 13.), the force of Gravity also on the summit of Pichincha must therefore be $\frac{1}{845}$ part less than the force of Gravity at the level

* Two corrections were applied for collecting this quantity from the immediate results of observation. 1st There must be an error in the comparative measurement of Pendulums in different temperatures, because the dimensions of the measure itself are varied. Hence the Pendulum in a colder temperature appeared longer than it really was, because its length was estimated by the divisions of a contracted measure. 2^{ly} The moving force of the Pendulum is not its whole weight, but the excess of its weight above the weight of an equal bulk of air. The lengths of the Pendulums therefore corresponded to forces less than the real forces of Gravity; and the diminution was unequal, the density, and consequently the weight, of the air being less in the more elevated situation. The effect of the former was to render the Pendulum in the more elevated situation, and consequently in the colder temperature, apparently longer than it really was; that of the latter was to render it really longer in comparison with the other, than it would have been, if the vibrations had been performed in a vacuum. La Figure de la Terre, sect. 7.

of

of the sea. The difference of these distances from the center of the earth, or 2434 toises, is $\frac{1}{1348}$ part of the radius of the earth. If therefore the force of Gravity were diminished in the reciprocal duplicate ratio of these distances, its diminution at the summit of Pichincha, or the correspondent diminution of the length of the Pendulum, should be $\frac{1}{674}$ part. The observations of these astronomers have not indeed discovered so great a diminution, but only that of $\frac{1}{845}$ part. But it is not difficult to account for the deficiency. A Pendulum vibrating on the summit of this mountain is not precisely in the same circumstances, as a Pendulum vibrating at a height of 2434 toises above a level surface. It will be shewn hereafter, that the attraction of a mountain is sufficiently sensible; and the Pendulum is therefore more powerfully attracted in the former than in the latter case. The observable diminution of the force of Gravity should therefore be * less, than it would be according to the general theory.

* The effect of the mountain in counteracting the diminution of Gravity must be greater, as it contains a greater mass of matter, or in proportion to its density. It might therefore be of such a density, as to counteract it altogether, and render the force of Gravity at its summit equal to the force at its base. Messrs Bouguer and De la Condamine have computed, from the actual diminution of Gravity, that the density of this mountain, notwithstanding the metallic substances which it contains, is not the fourth part of the general density of the globe of the earth. *Ibid.*

Scholium

Scholium 1. Hence, as has been already mentioned in page 36, we are enabled to collect that all the bodies of the planetary system gravitate towards each other. The revolutions of the primary planets round the sun, and of the satellites of any primary planet round the primary, are phenomena of the same kind as the revolution of the moon round the earth; and must consequently be considered as effects of similar forces: and though some primary planets have not satellites, yet we must consider them as bodies of the same kind with those which have satellites. Besides, by the 3rd law of motion, reaction is always opposed to action. We may therefore infer, that the sun reciprocally gravitates towards each primary, and * each primary, which is attended by satellites, towards the satellites which perform their revolutions round it. Farther: that the same kind of action takes place between the different primary planets, appears from the perturbations of the motions of Jupi-

* This is particularly confirmed in regard to the earth, by the phenomena of the tides. These have been shewn by Sir I. Newton (*Phil. Nat. Princ. Math. lib. 3. pr. 24*) to correspond in general, partly to a force directed towards the sun, but in a much greater degree to another directed towards the moon. The same force is also proved from the precession of the equinoxes, which has been also shewn to correspond to forces directed towards the two bodies, but principally to a force directed towards the moon. Sir I. Newton did indeed commit an error in this computation (*lib. 3. pr. 39*) making it by one half less than the truth; but D'Alembert and other mathematicians have given genuine solutions, which correspond to the phenomena, and the source of Sir I. Newton's error has been lately ascertained in a review of the solutions of this problem by Bishop Young, communicated to the Royal Irish Academy.

ter and Saturn near their conjunction; and that secondary planets are not only affected by forces directed towards their respective primary planets, but also by forces directed towards the sun, appears from the irregularities of the lunar motions, which have been * proved to correspond to the operation of such a force. The same

* This has been shown by Sir I. Newton in regard to all the lunar inequalities except one, which remained, until very lately, an acknowledged imperfection of the theory of Gravity. On this subject the following note has been communicated by Mr. Brinkley, Professor of Astronomy. "Dr. Halley first remarked, from the comparison of the Chaldean observations of lunar eclipses recorded by Ptolemy with those of Ptolemy himself and with modern observations, that the mean motion of the moon is greater at present than it was formerly. The researchers of later astronomers among the Arabian observations of the 10th century have ascertained the quantity of this acceleration. If the mean motion of the moon had been the same during the last 800 years as at present, the times of the eclipses of the 10th century would have been more than a quarter of an hour later than they are recorded to have been. This acceleration was long supposed to indicate a want of permanence in the solar system, since the increase of the moons motion seemed to prove that it was approaching nearer to the earth. At length Mr. De la Place, remarking that the united attraction of the Planets would, in the present situation of their nodes, have the effect of diminishing the excentricity of the earths orbit, computed the variation of the mean disturbing force of the sun occasioned by the variation of the excentricity; and found that this variation would produce an acceleration extremely near to that which had been observed, and that the acceleration was limited to a certain period, since, on account of the motion of the nodes, the effect of the attractions of the planets will be, after a very long period, an increase of the excentricity. It is worthy of remark, that the variation of the moons motion is much more apparent than the variation of the excentricity." This lunar inequality therefore affords the most complete proof of the reciprocal gravitation of the whole planetary system.

principle

principle of Gravitation has been applied by Sir I. Newton to the explanation of the phenomena of comets, which revolve round the sun in orbits of great excentricity, and consequently recede to distances far beyond the limits of the planetary system.

Schol. 2. From this reciprocal gravitation of all the bodies of our system, it follows that none of the bodies which compose it can be perfectly quiescent. A body which is urged by any force towards another, must either in obedience to that force move towards the other, or by its combination with a projectile velocity describe an orbit round it. But, since this is true of each body, neither can in strict propriety be considered as the absolute center of the motion of the other. That point only of the line joining their centers is quiescent, in which the opposite forces of the two bodies are balanced. It will be shewn, in treating of the Lever, that the distances of this point from the centers of the two bodies are in the reciprocal ratio of the quantities of matter which they contain. Each body must likewise be considered as having a center of Gravity, round which the Gravities of its parts are in equilibrium; and the common center of Gravity of two bodies is found by the division of a line conceived to join their respective centers of Gravity. These two being conceived to be united in their common center of Gravity, the common center of three bodies may be found, by dividing the line conceived to join the common center of the two with the center of the third, in the reciprocal ratio of the quantities of matter contained in the sum of the two bodies and in the third: and so in any number of bodies.

Schol. 3. The centripetal forces of the solar system being the same force of Gravity, which operates near the surface of the earth, the first law, which has been proved by experiment in regard to bodies falling near the surface of the earth, must be true of the planetary bodies in similar circumstances. The moving forces or weights of those bodies which revolve round a common central body, must therefore be proportional to their quantities of matter, when they are conceived to be at equal distances from the common central body; and the weights of any two bodies, as of a primary planet and its satellite, towards a third must also, at equal distances, be proportional to their quantities of matter. The same must likewise be true of the weights of the parts of any one body of the solar system towards any other.

Hitherto the force of Gravity near the surface of the earth has been considered as if it were of the same intensity in every part of that surface; but it has been discovered, that in different regions of the earth the forces of Gravity are sensibly different. Hence results a third law of Gravity.

Law 3. *The force of Gravity is least at the equator, and continually increases in receding from the equator towards either pole,*

It has been shewn (sect. 3. prop. 13.) that the lengths of Pendulums performing their vibrations in equal times are proportional to the forces of Gravity; and it has been uniformly observed, that the length of a Pendulum performing

performing each vibration in a second, is least at the equator, and gradually * greater in the countries more and more distant from it.

This variation is caused partly by the centrifugal force, which is derived from the rotation of the earth; and partly by the form of the earth which is probably the effect of that force, but is at least connected with it. The nature of this force must therefore now be explained.

Every body revolving round a center must, by the first law of motion, be influenced by a continual tendency to move forward uniformly in the direction of a tangent to the curve which it describes; and consequently, since every point of the tangent, except the contact, is exterior to the curve, to recede to a greater distance from the center of its motion. This tendency to recede from the center is called a centrifugal force. Since all the parts of the earth, except those which are in the axis, describe circles in the diurnal rotation, they must acquire such a tendency.

Prop. 1. The centrifugal forces of bodies revolving in circles with uniform velocities, are in the ratio compounded of the duplicate ratio of the velocities directly, and of the simple ratio of the radii reciprocally.

For the same spaces AD , ad (plate 2. fig. 5 and 6) or the respectively equal spaces EB , eb , which measure the

* The Pendulum is shorter at the equator than at Paris by $\frac{46}{100}$ line. La Figure de la Terre. sect. 7.

effects of the centripetal forces in A, a , do for the same reason measure also the effects of the centrifugal forces in the same points; and these spaces have been proved (sect. 3. prop. 3. cor.) to be in the above-mentioned compounded ratio.

Cor. If the revolutions be performed in equal times, the centrifugal forces will be proportional to the radii.

Since the times of the revolutions are equal, and the velocities uniform, the velocities must be proportional to the spaces described (page 18), or the peripheries, and consequently (Theor. fel. ex Arch. prop. 7. Tacq. Eucl.) to the radii. Since therefore, by the proposition, the centrifugal forces are in the ratio of $\frac{Vq}{R}$ to $\frac{vq}{r}$ (see page 44), the ratio of Rq to rq being substituted for the equal ratio of Vq to vq , the centrifugal forces must be in the ratio of $\frac{Rq}{R}$ to $\frac{rq}{r}$, or, by simplifying the fractions, in the ratio of R to r .

Prop. 2. The centrifugal force at the equator is to the centrifugal force at any parallel, the earth being supposed to be a sphere, in the ratio of radius to the co-sine of latitude.

In the diurnal rotation of the earth all the parts perform their revolutions round the axis in the same time, namely twenty four hours. Therefore, by the corollary of the preceding proposition, their centrifugal forces are proportional

proportional to the radii of the circles which they describe. Consequently (plate 4. fig. 1.) the centrifugal force at the equator E is to the centrifugal force at any parallel L , as CE to OL , or as CL to OL . But, since EL is the latitude, PL is the complement of latitude, and its sine OL is the co-sine of latitude, the radius of the earth CL being considered as radius.

Prop. 3. The whole centrifugal force at any parallel is to that part of it which directly counteracts Gravity, the earth being supposed to be a sphere, in the ratio of radius to the co-sine of latitude.

Let any part of OL , the radius of the parallel, produced, as LM , be assumed to represent the centrifugal force at L ; and on the radius of the earth CL produced let fall the perpendicular MN . The force LM is hereby resolved into two others represented by LN , NM , by the 4th cor. of the 2nd law of motion. Of these NM , being perpendicular to the direction of Gravity, which tends towards the center C , produces no effect in regard to that force; and LN , which tends directly from the center, is therefore that part of LM the centrifugal force at L , which counteracts Gravity. But, on account of the similarity of the triangles MLN , CLO , $LM : LN :: CL : OL$, that is, as radius to the co-sine of latitude.

Prop. 4. The effect of the centrifugal force in diminishing Gravity at the equator, is to its effect in any parallel, in the duplicate ratio of radius to the co-sine of latitude, the earth being supposed to be a sphere.

For

For at the equator the whole of the centrifugal force is opposed to Gravity, because it tends directly from the center of the earth. Since therefore (prop. 2.) the centrifugal force at the equator is to the whole centrifugal force at any parallel, as radius to the co-sine of latitude; and the whole centrifugal force at the parallel is to that part of it which operates against Gravity in the same ratio (prop. 3.); the effect of the centrifugal force in diminishing Gravity at the equator, must be to its effect at the parallel, in the ratio compounded of these two ratios, or in the duplicate ratio of radius to the co-sine of latitude.

Scholium. From these propositions it appears, that in all parts of the surface of the earth, except the poles, a force is exerted, which is in a greater or less degree

• The quantity of this centrifugal force would be increased, if the velocity of rotation were increased (prop. 1.); and if the acceleration were so great, that the time of the earth's rotation should be only 1 hour 24 minutes 27 seconds, all the parts at the equator would wholly lose their Gravity, and describe circular orbits round the center of the earth.

To prove this it should be considered, that a body will revolve in a circle if it be projected with a velocity equal to that, which it would acquire in falling down the half of the radius with the centripetal force continued constant (see note in page 48). There is therefore a certain acceleration which would produce the effect. But in this case the arch described in a second, being considered as coinciding with its chord, is a mean proportional between the space through which Gravity causes a body to descend in the same time, or the versed sine of the arch, and the diameter of the orbit or of the earth. Since then that

space

directed opposed to Gravity. At the equator this force is greater than at any parallel (prop. 2); and at the equator it is also exerted in direct opposition to Gravity, whereas its opposition becomes more and more oblique in receding towards the pole (prop. 3). On account of these two reasons combined together, it is found (prop. 4) that the force which is exerted in diminution of Gravity at the equator, is to the force at any parallel, in the duplicate ratio of that of radius to the co-sine of latitude. The force of Gravity must therefore be most diminished at the equator, and gradually less diminished in receding towards the pole; that is, it must be least at the equator, and gradually increase in receding towards the pole.

But there is * another cause of the variation of the force of Gravity in different parts of the surface of the

M

earth

space is known (lemma to law 2), and the diameter of the earth is also known, the arch which should be described in a second, is found to be nearly 5 miles. But with such a velocity the whole periphery of the equator would be described in 1 hour 24 minutes 27 seconds. With a greater velocity the same parts of the surface would describe eccentric orbits.

Though the rotation is not sufficiently rapid to produce this effect, yet it must inevitably hinder a falling body from really descending in a straight line, since such a body is in fact projected with the velocity of the surface. A body never really descends in a straight line, except when it has been projected with an equal velocity in the opposite direction.

* In the account of the observations of Messrs. Bouguer and De la Condamine, it is remarked that the centrifugal force is of itself inadequate to the diminution of Gravity at the equator. The quantity

of the centrifugal force at the equator was found to be $7 \frac{54}{100}$ lines

by

earth, besides the direct operation of the centrifugal force which has been considered. The earth has hitherto been regarded as a perfect sphere, but it is certain that its surface is at the equator more distant from the center than at the poles. This has been ascertained by observing, that a degree of a meridian at the equator corresponds to a smaller measured space on the surface, than a degree in countries near the pole. The meridian is therefore not a circle; and since a degree is of less magnitude at the equator, it must there belong to a curve of greater curvature, as being similar to the curvature of a smaller circle. The surface is consequently protuberant

by computing the versed sine of the arch described in a second by the rotation of the earth (see prop. 1). As this force is exerted in opposition to Gravity, this space should be added to the space determined by the Pendulum for the motion produced by Gravity in a second (lemma to law 2); since, if not counteracted in part by the centrifugal force, Gravity would in the same time cause a body to descend through so much a longer space, by the 2nd property of motions produced by constant forces. Therefore (sect. 3. prop. 13.) the length of the Pendulum at the equator should be increased in the same ratio,

and the difference would be $1\frac{53}{100}$ line. From this was collected (prop. 4.) the corresponding augmentation of the Pendulum at the latitude of Paris, as being proportional to the correction of Gravity, and this was found to be $\frac{67}{100}$ of a line. The diminution of the observed Pendulum arising from the variation of the centrifugal force is therefore only the difference of those two corrections, or $\frac{86}{100}$

of a line. But the observed difference is $1\frac{46}{100}$ line. The centrifugal force therefore accounts only for about $\frac{3}{5}$ of the diminution. La Figure de la Terre. sect. 7.

at the equator, and flattened at the pole; and therefore more elevated in the former region than in the latter †.

If the whole earth has been at any time in a fluid state, this form must have been the effect of its rotation; since the centrifugal force, by causing a variation in the force of Gravity at equal distances from the center, would disturb the equilibrium of the parts of the earth, and those parts, which were less powerfully pressed by Gravity, would ascend until the accumulation of matter should compensate the diminution of Gravity, and the different parts of the earth should be restored to equilibrium. Even though the greater part of the earth had been originally solid, as in its present state, yet this elevation at the equator must have been given to it, that the

† Messrs. Bouguer and De la Condamine found that a degree at the equator contained 56753 toises, whereas a degree measured in France contained 57074, and a degree at the Polar circle 57422. From these observations they computed that the diameter of the equator is to the axis of the earth as 179 to 178. *La Figure de la Terre*, sect. 6. Sir I. Newton had, on the supposition that the earth was of an uniform density, computed that the diameter of the equator is to the axis of the earth only as 230 to 229. *Phil. Nat. Princ. Math.* lib. 3. prop. 19. We must therefore conclude, that the density of the earth is greater near the center than near the surface.

Sir I. Newton has endeavoured (*Phil. Nat. Princ. Math.* lib. 3. prop. 26.) to determine the ratio, in which the force varies in different parts of the surface; but his solution rests on two assumptions, neither of which can safely be adopted. The one is that the density of the earth is uniform; the other that the meridian is an ellipse. The law which he has assigned for the diminution of Gravity within the surface (*ibid.* prop. 9.) namely the simple ratio of the distances from the center, depends on the former of those suppositions.

seas should not, in consequence of the variation of Gravity, subside at the poles, and inundate the equatorial regions.

This inequality of the distances of different parts of the surface from the center is an additional cause of the variation of Gravity at the surface. A body placed at the equator is attracted by the same mass of matter, which would attract it at the pole; but it is evidently in a situation less advantageous for experiencing the influence of its force. Its situation resembles that of a body on the summit of a lofty ridge of mountains. Its attraction is diminished on account of its increased distance, but not so much diminished as it would be if it were placed at the same elevation above a level surface.

One other law remains to be considered, which concerns Gravity regarded as a force not merely tending towards the earth, but also towards each of the bodies which compose the solar system. Though the forces directed towards these bodies are of the same kind, yet they differ in intensity; and it is the object of this last law to compare them in this respect.

Law 4. The accelerating forces of Gravity directed towards different bodies are proportional to the quantities of matter, which they contain.

It has been shewn (Ichol. to law 2.) that each body of the system attracts all the others, and that each of these attracting forces varies in the reciprocal duplicate ratio of the

the distances from the attracting body. If now A , B , C , D , &c. represent the bodies of such a system, the accelerating attractions of all the bodies B , C , D , towards A would at equal distances be equal; and in the like manner the accelerating attractions of A , C , D , towards B would also at equal distances be equal. Whatever ratio therefore the accelerating force of any one of the former towards A has to the accelerating force of any one of the latter towards B at an equal distance, the same is the ratio of the intensities of the attracting forces of the bodies A and B . But B is one of the former, and A is one of the latter; and, since the reciprocal actions of these bodies are equal, by the 3rd law of motion, the moving force or weight by which B is impelled towards A is equal to the moving force by which A is impelled towards B . Since therefore this equal ratio is, by the scholium to the 3rd law of motion, compounded of the ratios of the velocities or accelerating forces and of the quantities of matter, these ratios must be reciprocal. Consequently the accelerating force of B towards A is to the accelerating force of A towards B , as the quantity of matter in A to the quantity of matter in B ; and the former ratio has been already shewn to be the ratio of the intensities of the forces directed towards A and B .

Moreover, since (schol. to the 2nd law of Gravity) the weights of the several parts of A towards B are proportional to their quantities of matter, and reaction is equal to action (3rd law of motion); the attractions of B towards the several parts of A must be proportional to their

their quantities of matter. What therefore had been before proved of the intire bodies, is true also of their component parts.

This latter position has been confirmed by repeated experiments. Though the mass of matter contained in any mountain is very inconsiderable when compared to the whole earth, yet the opportunity of bringing a body near to its center compensates in some degree this inferiority, and renders its attraction sensible. The French Academicians accordingly observed that by the attraction of the mountain Chimboraco a plummet was drawn * $7\frac{1}{2}''$ from the perpendicular direction; and a similar

* They computed that the deviation should have been $1'. 43''$. The difference they ascribed to the deficiency of the density of the mountain. In this opinion they were confirmed by its volcanic appearance, which rendered it probable that much of it was hollow. The experiment was made by observing the meridian altitudes of the sun or of stars in two places situate east and west, one near the center of the mountain, the other at such a distance that the attraction must be insensible. From the difference of the observations was collected the deviation of the plummet. They were however of opinion, that an addition of $\frac{1}{13}$ or

$\frac{1}{14}$ part should be made to the observed deviation of $7\frac{1}{2}''$; because, on a comparison of the distances of the two stations from the center of the mountain, they concluded that even at the more distant station an attraction was exerted, and consequently the observed effect of the attraction was only the difference of the two effects, and to give the whole should be augmented by the addition of the lesser. The attraction at the nearer station should consequently be estimated at $8''$.
La Figure de la Terre. sect. 7.

The

milar experiment made in Scotland discovered that the mountain Schehallion attracted the plummet from its direction $5''$, $33'''$.

The force of Gravity which is thus extended through the whole solar system, is, as * Dr. Herschell conceives, diffused throughout the whole universe. To prove that all the stars are subject to its influence, he first enumerates the consequences which would probably follow from their mutual gravitation, and then compares these consequences with the actual appearances observa-

The experiment was made in Scotland by observing the meridian zenith-distances of different stars near the zenith from two stations, of which the one was on the south, the other on the north side of the mountain. The apparent zenith-distances were affected in opposite ways, those being increased at the one station which were diminished at the other; and the correspondent deviations of the plummet were the effects of the sum of the two attractions. The half therefore, which was $5''$, $33'''$, indicated the attractive power of the mountain. If its density had been equal to the mean density of the earth, the effect of the attraction should have been about double of this quantity. Since then the mountain does not appear to have ever been volcanic, and seems to be composed of an intire rock, it may be concluded that the mean density of the earth is at least double of that at the surface, and consequently that the density of the internal parts of the earth is considerably greater. This conclusion, Mr. Maskelyne observes, is totally contrary to the hypothesis of some naturalists, who "suppose the earth to be only a great hollow shell of matter; supporting itself from the property of an arch, with an immense vacancy in the midst of it." Phil. Trans. vol. 65. part. 2.

In a very accurate computation founded on these observations by Mr. Hutton, it was determined that the mean density of the earth is to that of the mountain almost as 9 to 5. Phil. Trans. vol. 68. part. 2.

* Phil. Trans. vol. 75. part. 1.

ble among the stars. "Let us then suppose," says he, "numberless stars of various sizes scattered over an indefinite portion of space in such a manner as to be almost equally distributed throughout the whole.

The laws of attraction, which no doubt extend to the remotest region of the fixed stars, will operate in such a manner as most probably to produce the following remarkable effects.

1. Since we have supposed the stars to be of various sizes, it will frequently happen that a star, being considerably larger than its neighbouring ones, will attract them more than they will be attracted by others that are immediately around them; by which means they will be in time, as it were, condensed about a center; or, in other words, form themselves into a cluster of stars of almost a globular figure, more or less regularly so, according to the size and original distance of the surrounding stars.

2. The next case, which will also happen almost as frequently as the former, is where a few stars, though not superior in size to the rest, may chance to be rather nearer each other than the surrounding ones; for here also will be formed a prevailing attraction in the combined center of Gravity of them all, which will occasion the neighbouring stars to draw together; not indeed so as to form a regular or globular figure, but however in such a manner as to be condensed towards the common center of Gravity of the whole irregular cluster. And
this

this construction admits of the utmost variety of shapes, according to the number and situation of the stars which first gave rise to the condensation of the rest.

3. From the composition and repeated conjunction of both the foregoing forms a third may be derived, when many larger stars, or combined small ones, are situated in long extended, regular, or crooked rows, hooks, or branches; for they will also draw the surrounding ones, so as to produce figures of condensed stars coarsely similar to the former which gave rise to these condensations.

4. We may likewise admit of still more extensive combinations; when, at the same time that a cluster of stars is forming in one part of space, there may be another collecting in a different, but perhaps not far distant quarter, which may occasion a mutual approach towards their common center of Gravity.

5. As a natural consequence of the former cases, there will be formed great cavities or vacancies by the retreat of the stars towards the various centers which attract them—

At first sight it will seem as if a system, such as it has been displayed in the foregoing paragraphs, would evidently tend to a general destruction, by the shock of one star's falling upon another. It would here be a sufficient answer to say, that if observation should prove this really to be the system of the universe, there is no doubt but that the great Author of it has amply provided for

the preservation of the whole, though it should not appear to us in what manner this is effected. But I shall moreover point out several circumstances, that do manifestly tend to a general preservation; as, in the first place, the indefinite extent of the sidereal heavens, which must produce a balance that will effectually secure all the great parts of the whole from approaching to each other. There remains then only to see how the particular stars belonging to separate clusters will be preserved from rushing on to their centers of attraction. And here I must observe, that though I have before, by way of rendering the case more simple, considered the stars as being originally at rest, I intended not to exclude projectile forces; and the admission of them will prove such a barrier against the seeming destructive power of attraction, as to secure from it all the stars belonging to a cluster, if not for ever, at least for millions of ages. Besides, we ought perhaps to look upon such clusters, and the destruction of now and then a star, in some thousands of ages, as perhaps the very means by which the whole is preserved and renewed. These clusters may be the *Laboratories* of the universe, if I may so express myself, wherein the most salutary remedies for the decay of the whole are prepared."

From this theoretical view of the heavens Dr. Herschell proceeds to describe the appearances, which they actually exhibit. "I shall now endeavour to shew," says he, "that the theoretical view of the system of the universe, which has been exposed in the foregoing part of this paper, is perfectly consistent with facts, and seems

to

to be confirmed and established by a series of observations. It will appear, that many hundreds of nebulae of the first and second forms are actually to be seen in the heavens. Many of the third form will be described, and instances of the fourth related. A few of the cavities mentioned in the fifth will be particularised, though many more have already been observed; so that, upon the whole, I believe, it will be found, that the foregoing theoretical view, with all its consequential appearances, as seen by an eye inclosed in one of the nebulae, is no other than a drawing from nature, wherein the features of the original have been closely copied; and I hope the resemblance will not be called a bad one, when it shall be considered how very limited must be the pencil of an inhabitant of so small and retired a portion of an indefinite system in attempting the picture of so unbounded an extent."

This subject Dr. Herschell has further considered in a Memoir contained in the Phil. Trans. vol. 79. part 2. in which he has inferred from the spherical form which the nebulae or clusters generally affect, that they must be formed by attraction in the same manner in which that cause gives the same form to the bodies of the planetary system; and has confirmed this inference by remarking a gradual condensation towards the center of each nebula.

CHAP. II.

OF CORPUSCULAR

ATTRACTION AND REPULSION.

THE phenomena which have been considered in the preceding chapter, are capable of very accurate determination, because they are produced by a force which acts between the great masses of the universe, and is extended through all distances. These operations do indeed on account of their magnitude demand the exertion of the utmost powers of our understandings, and acquire a relative subtilty from the remoteness of their objects; but they do not much embarrass us by intricacy of combination. The operations which must now be investigated, are of a very different kind. The phenomena which immediately surround us, the compositions and decompositions of all substances animate and inanimate, and even that cohesion which gives to solid bodies their form and

and permanency, are fubjects of the moft minute and complicated inquiry. The full profecution of this inquiry is the object of Chemistry, as it has been defined by Mr. Houtbroy to be "that fciences, which explains the intimate mutual action of all natural bodies." In this elementary view of the phenomena of nature it will be fufficient to describe fuch obvious effects, as prove this intimate mutual action of the parts of bodies; and to point out the general laws, which have been obferved to regulate it.

That the minute particles of matter, when placed at fmall diftances are attracted towards each other, is evident from many confiderations. That this force exifts between the parts of folid bodies appears, 1st from the neceffity of exerting a force for feparating the parts of one and the fame body; and 2^{ly} from the difficulty of feparating two polished fufaces, which have been brought into clofe contact. This latter obfervation is exemplified, "fays Mr. Nicholfon, " by paring a fmall part from each of two leaden bullets, and preffing the fufaces together; in which cafe, with a furface of contact not exceeding the twentieth part of a fquare inch, it will frequently require the force of 100 lbs. to feparate them."

That the parts of fluids are influenced by this force, may be inferred from the fpherical form, which their drops affect. A fphere is that form, which contains the greateft quantity within a determinate furface (Theor. fel. ex Arch. prop. 4. cor. 4.); and thofe particles which

* Introduction to Natural Philofophy. vol. 1. p. 46.

affect

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affect this form, must be urged by some force of mutual attraction. The attraction subsisting between the parts of solid and of fluid bodies is evinced by various phenomena. 1st. The adhesion of fluid to solid bodies proves that the former are attracted towards the latter: 2^{ly} if water be contained in a wide vessel not full, it will rise at the sides of the vessel: 3^{ly} if two glass planes be immersed in water in a vertical position and at a very small distance, the water will be elevated between them above the level of the external water; and 4^{ly} the same effect will be produced, if a capillary tube, or a tube of a very small bore, be immersed in water, both ends of the tube being open.

In regard to the phenomena mentioned in the preceding paragraph it must be observed, that the cohesion of the parts of the same solid body cannot be the effect of their inertness; since, the inertness of a part being less than that of the whole, less force would be requisite for separating a part from the remainder than for removing the whole body from its place: and that the others cannot be the effects of the * pressure of the atmosphere, since they are observed to happen in the same manner in vacuo.

In the phenomena of the planes and capillary tubes the equilibrium of the fluid will preserve it at the same

* It has indeed been maintained, that the elevation of water in very slender capillary tubes is assisted by the comparative rarity of the air included within the tube. The water situate immediately beneath the orifice, if less pressed by the incumbent air, must ascend to a small height until an equilibrium of pressure shall have been effected.

height

height between the planes or within the tubes, which the external water possesses, whatever may be the depth to which the planes or tubes are immersed. The elevation above this level is the effect of the attraction of those parts of the glass, which are situate immediately above the surface of the water; and consequently of different parts as the water continues to ascend. The water must continue to ascend until the weight of the elevated portion is equal to the force of the attraction of the glass.

* No other parts of the glass than those which have been mentioned can contribute to the elevation of the water. It is obvious that those parts which are on the same level as the water, cannot assist in elevating it; and those which are beneath that surface, must exert the same force in depressing the superior part of the water, which they exert in raising the inferior, and must consequently be wholly inoperative.

The water is observed to ascend higher in tubes of smaller diameters. This may be explained partly by a

* It has been thought, and with great appearance of probability, that the elevation of the water in a capillary tube, and the same reasoning must be applicable to the planes, is the sole effect of the attraction of the parts at the lower orifice. That this however is not the cause of the phenomenon appears from an experiment mentioned in Rowning's Compendious System of Natural Philosophy, vol. 1. p. 138.

If a capillary tube consist of two parts, whose diameters are different, the elevation of the water when the wider part is immersed, will be the same as if the whole tube were of the smaller diameter; but when the narrower part is immersed, the same as if the whole were of the larger diameter. The elevation of the water depends therefore on the attraction of the upper part of the tube.

consideration

consideration of the cylindrical form of the elevated water, and partly by a consideration of the increased energy of the attractive power in the narrower tubes.

The attracting forces of different tubes, being the forces of those rings of glass which are situate immediately above the elevated water, must be proportional to the magnitudes of those rings; and consequently, since the breadths of the attracting rings are determined by the distance at which the force operates, and are therefore equal in both tubes, they must be proportional also to the diameters of the rings (Theor. 1. ex Arch. prop. 10. cor. 2.) or of the tubes. The cylinders of elevated water must accordingly be proportional to the diameters of the tubes, as being proportional to the forces by which they are elevated. But, that the contents of two cylinders should be in the ratio of their diameters, * it is necessary that the altitudes should be reciprocally as the diameters. The water must therefore ascend to a greater altitude in the smaller tube.

But † it has been observed, that the water ascends in

* Any two cylinders are in the ratio compounded of the direct simple ratio of their altitudes, and of the direct duplicate ratio of their diameters (Eucl. book 12, prop. 15. schol. 1.): but, in order that this compound ratio should be equivalent to the simple ratio of their diameters, it is necessary that the simple ratio of their altitudes should reduce the duplicate ratio of their diameters to a simple ratio; that is, that it should counterbalance one of the two simple ratios which form the duplicate, or that the ratio of the altitudes should be the reciprocal ratio of the diameters.

† Four Introductory Lectures in Natural Philosophy. p. 41.

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the narrow tubes to greater heights than would result from the reciprocal ratio of the diameters. The cause of this must be the increased energy of the force of such tubes arising from the greater proximity of the attracting particles. That the force is thus augmented appears from the following experiment. Let two well polished planes of glass be laid together, so that they may make a small angle, and that their concourse may be parallel to the horizon; and, their inner sides being moistened with any thin oil, let a drop of the same oil be introduced, so that it may touch both planes. The directions of the attractive forces of the planes, being perpendicular to them, will form an angle on the side towards their concourse, and therefore, by the 1st corollary of the 2nd law of motion, compound a force, which will attract the drop towards the concourse. By elevating the concourse of the planes sufficiently above the other extremity of the lower plane, the drop may be stopped at any distance from the concourse; and by doing this when the drop is at different distances, the attractions exerted upon the drop at those distances may be compared together. For the drop must be considered as a body supported on an inclined plane, and, since it is stopped, the attracting force which draws it upwards,

* This must be a consequence of the equality of the attracting forces of the several points of the planes. For the points perpendicularly opposed to the drop will draw it perpendicularly; and, on account of the equality of the attracting forces, those of the points equally distant from each of these on opposite sides will be equally exerted upon the drop, and, by the 1st corollary of the 2nd law of motion, compound an intermediate force in a perpendicular direction.

must in each instance be equal to that part of its weight by which it endeavours to descend. But (ch. 1. sect. 3. prop. 1.) the force of descent is in each instance to the whole weight, as the height of the plane to its length; and therefore, the weight of the drop and the length of the plane being unvaried, the forces of descent are proportional to the perpendicular heights of the concourse at its different elevations. The ratio of these heights consequently is also the ratio of the attractions exerted upon the drop at the several distances from the concourse; and this ratio is found by experiment to be the reciprocal-duplicate ratio of those distances. This ratio must however be reduced to the reciprocal simple ratio of the distances, because the augmentation of force in the other reciprocal simple ratio is occasioned by the spreading of the drop, which brings it into contact with larger portions of the attracting surfaces.

The phenomena of capillary tubes have been examined by philosophers with much attention, because they have been supposed to have an affinity to the ascent of the sap through the stems of plants, and to the operation of the glands in the bodies of animals. But, besides that physiologists assign another cause of the passage

* The drop may be considered as nearly cylindrical because the planes, by which it is terminated, are nearly parallel. Since therefore cylinders of the same magnitude have their bases reciprocally proportional to their altitudes, (Euch. book 12. prop. 15.) the portions of the planes, which are at the different distances from the concourse contiguous to the drop, must be reciprocally proportional to the intervals of the planes at those distances, and consequently to the distances themselves.

of

of fluids through the vessels of organized bodies, namely an alternate expansion and contraction of the vessels, the fluids themselves acquire new properties. "If plants," says * M. Chaptal, "were to perform no other act than that of pumping the nutritive principles they contain out of the earth; if they did not possess the faculty of digesting, assimilating them, and forming different products, according to their nature, and the diversity of their organs; it would follow, as a consequence, that we ought to find in the earth all those principles which analysis exhibits to us in vegetables: a conclusion which is contradicted by the facts." And, speaking of the imperfect success of chemists in the application of their science to medicine, he † observes that "all have mistaken or overlooked that principle of life, which incessantly acts upon the solids and fluids; modifies, without ceasing, the impression of external objects; impedes the degenerations which depend on the constitution itself; and presents to us phenomena, which chemistry never could have known or predicted by attending to the invariable laws observed in inanimate bodies."

‡ Corpuscular attraction, or that attraction which
O 2 acts

* Elements of Chemistry. part 4. sect. 2.

† Ibid. part 5. introd.

‡ Mr. Nicholson in his Dictionary of Chemistry (Art. Attraction) remarks, that "the philosophers at the beginning of the present century were disposed to consider the several attractions as essentially different, because the laws of their action differ from each other; but the moderns appear disposed to generalize this subject, and to consider all the attractions which exist between bodies, or at least those which are permanent,

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acts between the minute particles of bodies, is divided into two kinds: 1. the attraction of aggregation, or that which exists between two substances of the same nature; 2. the attraction of composition, which retains two or more substances of different natures in a state of combination.

The attraction of aggregation exists in various degrees, and may be wholly counteracted; and there are accordingly reckoned three different states of aggregation, solidity, fluidity, and vapour. Solidity is that state, in which the integrant parts are united by a considerable force, and cannot be separated without some exertion. Fluidity is that, in which the integrant parts are so

slightly permanent, as depending upon one and the same cause, whatever it may be, which regulates at once the motions of the immense bodies which circulate through the celestial spaces, and those minute particles which are transferred from one combination to another in the operations of chemistry. The earlier philosophers observed, for example, that the attraction of gravitation acts upon bodies with a force which is inversely as the squares of the distances; and from mathematical deduction they have inferred, that the law of attraction between the particles themselves follows the same ratio: but, when their observations were applied to bodies very near each other, or in contact, an adhesion took place, which is found to be much greater than could be deduced from that law applied to the centres of gravity. Hence they concluded, that the cohesive attraction is governed by a much higher ratio, and probably that of the cubes of the distances. The moderns, on the contrary, among whom are Bergmann, De Morveau, and others, have remarked that these deductions are too general, because for the most part drawn from the consideration of spherical bodies, which admit of no contact but such as is indefinitely small, and exert the same powers on each other, whichever side may be obverted. They remark

slightly united, that the gentlest effort is sufficient, not only to change their relative situation, but even to divide them into distinct globules. Vapour is that, in which the tenacity of the integrant particles renders them imperceptible, and, instead of attracting, they repel each other.

The attraction of aggregation is diminished by any cause, which increases the distances between the integrant parts. It is in this manner diminished by the mechanical operations of pounding, or cutting. Heat, which

mark likewise, that the consequence depending on the sum of the attractions in bodies not spherical, and at minute distances from each other, will not follow the inverted ratio of the square of the distance taken from any point assumed as the centre of gravity, admitting the particles to be governed by that law; but that it will greatly differ, according to the sides of the solids which are presented to each other, and their respective distances; insomuch that the attractions of certain particles indefinitely near each other will be indefinitely increased, though the ratio of the powers acting upon the remoter particles may continue nearly the same."

The following arrangement of the objects of Corpuscular Philosophy has been adopted since the year 1782 by Dr. Perceval, Professor of Chemistry.

Action of Particles.	Without permanent change of properties.	Superficial.	1. Superficial adhesion. 2. Electricity. 3. Optics.
		Internal.	1. Aggregation. 2. Crystallization. 3. Internal cohesion. 4. Magnetism.
	With permanent change of properties.		1. Chemistry, which relates to inanimate bodies. 2. Physiology, which relates to animated bodies.

expands

expands the dimensions of all bodies, must, for the same reason, weaken or destroy the attraction of aggregation; and accordingly many solids are by the application of heat successively converted into the states of fluidity and vapour. But if heat be itself a fluid, the changes of aggregation which it occasions, are rather instances of the 2^d species of Corpuscular Attraction; since they must in this case be produced by the combination of the particles of bodies with a * greater or smaller portion of that fluid.

Heat, or, according to the nomenclature of some modern chemists, Caloric, denotes in its philosophical use the cause of the power which bodies possess of exciting the sensation of heat or coldness. The relative state of any body in regard to our sensation of heat or coldness is called Temperature. Thus a body which excites a sensation of considerable heat, is said to be of an high temperature. Two opinions have been entertained concerning the nature of heat, or of the cause of the sensation: one that it consists merely in a motion excited in the particles of a body; the other that it is a fluid combined with its particles, which becomes sensible when it is extricated from them. The former opinion rests

* The vapours into which bodies are expanded by heat, are only distinguishable from the permanently elastic fluids usually denominated Airs, by this criterion that the latter are not condensable into the liquid state by any degree of cold hitherto produced. The air of the atmosphere was formerly thought to consist of one uniform fluid exclusively possessing the permanently elastic state. Modern philosophers, among whom Dr. Priestley may be styled the father of this branch of philosophy, have discovered a considerable number of different kinds of air.

principally

principally upon the observation, that the temperature of a body may be raised by percussion and friction, which it is supposed, can only cause a greater agitation of its parts. The latter is a deduction from a number of recent discoveries concerning the nature of heat.

Two laws of temperature have been observed to obtain among bodies.

1. An increase of temperature effects an * increase of dimension.

In a given increase of temperature fluids are more expanded than solids, and vapours more than fluids. Some solids are more expanded in the same increase of temperature than others. It has been conjectured, that this may arise from their inflammability; since metallic substances are observed to be expanded more than other solids, and inflammable air is found to be capable of very considerable expansion. The vapour of water, or steam, has been very advantageously applied to mechanical purposes. Its sudden expansion and condensation enable us alternately to apply and to withdraw a very powerful force; and thereby to give motion to a machine, which has been hence denominated a Steam-engine.

2. If several bodies of different temperatures be brought together, they will acquire a common temperature of some intermediate degree.

* From the expansions and contractions caused by variations of temperature it has been inferred, that the particles even of bodies in a solid state are not actually in contact.

Fluids are observed to attract heat more than solids, and vapours more than fluids. That the conducting powers of bodies in the same state of aggregation are different, is proved by coating with wax similar rods of metal, wood, and glass, and plunging them in the same heated fluid. It is found, that in the same time the wax on different rods will be melted to different heights above the surface of the fluid. Count Rumford has lately * maintained an opinion, that all fluids are essentially non-conductors of heat; and his experiments prove, that they transmit it by the actual motion of their particles, whose comparative weight is diminished by expansion, and that every method of obstructing or retarding this motion would render the propagation of heat more slow and difficult. The motions of very fine particles of dust accidentally mixed with spirits of wine strongly illuminated by the sun, first led him to the discovery of the internal motions, which take place in that fluid whilst it is cooling. An ascending current was seen to occupy the internal part of the tube which contained the spirits of wine, and a descending current was contiguous to the sides. The same motions appeared in linseed oil; and the Count contrived to render them visible in water, by introducing into it some particles of amber, which the water was enabled to bear by the addition of a certain quantity of pure alkaline salt. That the obstruction of these motions will render the communication of heat more difficult, he proved by placing a tube containing linseed oil within an hollow cylinder containing the

fluid whose conducting power was to be tried; and in this manner he found, that the conducting power of water was considerably diminished, both when its fluidity was diminished by a solution of any mucilaginous substance as starch, and when merely the motion of its particles was impeded by mixing with them any solid substance, which is an imperfect conductor of heat, as eider-down. The heat communicated was estimated by the expansions of the linseed oil. It is however remarked by * Mr. Nicholson, that "if each individual particle were not capable of receiving and giving heat, the process by circulation could not happen; and if on the contrary they be capable, the propagation of heat from particle to particle is possible, and must doubtless happen; though from the facts it appears that the heat conveyed by the internal motion is very much greater than what passes in this way. Fluids then are very imperfect conductors of heat, but not perfect non-conductors." This discovery of the nature of fluids has been happily applied by the Count to the illustration of the economy of nature. He remarks, that the extreme smallness of the vessels in which the sap moves in vegetables, and particularly in large trees; the substance of which these small tubes are formed, which is one of the best non-conductors of heat known; the increased viscosity of the sap in winter; and the almost impenetrable covering for confining heat, which is formed by the bark; account for the preservation of the requisite degree of warmth in trees during the winter, notwithstanding the long continuance of a degree of cold more than sufficient to freeze water not guarded by these pre-

* Journal of Nat. Philosophy, &c. for October 1797.

cautions. He farther remarks, that those trees, which brave the cold of the severest winters, contain very little liquids, and that their sap is thick and viscous. Count Rumford had before discovered, that air is a bad conductor of heat; and explained the warmth of woollen cloathing on the principle, that by strongly attracting the particles of air, it hindered the successive application of new particles to the body, and consequently prevented the successive abstraction of new portions of heat, whilst the air which remained contiguous to the body, was unable to transmit it copiously to the adjacent particles.

These two laws of temperature enable us to construct an instrument, called a Thermometer, by which the temperatures of bodies are compared. Since the sensation of heat depends partly on the state of the organ of sensation, which is liable to change, it cannot be considered as an accurate standard of temperature. It is therefore necessary to contrive an instrument, which should exhibit variations of temperature, such as they would be felt if our organs remained unaltered. From the 2^d law we learn, that a substance applied to another will come to a common temperature with it; and from the 1st, that its dimensions will vary with the changes of its own temperature. All then that is requisite is to contrive, that the instrument should not sensibly vary the temperature of the body to which it is applied, so that its acquired temperature may be considered as the original temperature of the body; and that its expansions and contractions should be easily perceptible. The former is effected

effected by giving a small bulk to the instrument; the latter by including a fluid in a bottle, whose * neck is long and very narrow. The thermometer accordingly consists of a glass ball with a long narrow tube, which is † partly filled with mercury. This fluid is preferable to all others, because it is unchangeable, expands with ‡ regularity, and does not soil the tube.

For graduating a thermometer it is necessary to ascertain two degrees of temperature, by which the scale may be determined. The two thermometers most frequently used agree in the choice of one of these, but differ in regard to the other. The temperature of § boil-

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* It may be ascertained whether the tube is exactly cylindrical, by including in it a small quantity of mercury, and observing whether, when it is moved to different parts of the tube, it occupies equal lengths.

† It is desirable that the air should be perfectly excluded from the upper part of the tube, that it might not disturb the expansions of the mercury. In the construction of thermometers it is almost wholly excluded by the application of heat.

‡ The experiments, by which the regularity of the expansions of mercury is proved, are detailed by Dr. Crawford in his *Treatise on Animal Heat*. 2nd Ed. The immediate result of those experiments, which were first made by M. De Luc, is that the expansion of a mercurial thermometer immersed in a mixture of equal quantities of hot and cold water, or of linseed oil, or exposed to two equal metallic surfaces of different temperatures, was very nearly the arithmetical mean between the expansions which it had in the hot and cold fluids before the mixture, or in the separate temperatures of the surfaces. Hence it is inferred, that the expansions of the mercury correspond to the variations of temperature.

§ The degree of heat requisite for causing water to boil, depends partly on the pressure of the atmosphere, which resists its evaporation.

This

ing water was adopted both by Reaumur and Fahrenheit as one point of the scale; but Reaumur chose to begin his scale from the temperature of freezing water, whilst Fahrenheit computed his from a still lower temperature produced by a mixture of snow and sal ammoniac. The temperature of freezing water, which is marked 0 in Reaumur's scale, is accordingly marked 32 in that of Fahrenheit. But these scales differ also in their divisions. In Reaumur's scale only 80 degrees are counted between the temperatures of freezing and boiling water: but in Fahrenheit's scale * 180, and consequently the temperature of boiling water is marked 212.

The mercurial measures of temperature are naturally limited by that of boiling mercury, which is at the 600th degree of Fahrenheit's scale; but a measure of still

This temperature should therefore be found at a certain known height of the barometer. For this purpose the height of 29.8 inches has been chosen by the Royal Society. The pressure of the atmosphere on the outside of a thermometer, not being counteracted by the spring of any included air, is exerted in diminishing the size of the ball, and consequently sustains the mercury at a somewhat greater height, than that at which it would stand merely on account of its temperature. This is proved by breaking off the closed end of the tube, in consequence of which the mercury immediately falls. This quantity of additional elevation varies with the weight of the atmosphere, but the quantity of the variation can seldom amount to more than the tenth part of a degree. Thermometers with balls are less affected by the atmosphere, than those with bulbs of other shapes.

* To reduce these scales to each other it must be observed, that the number of degrees of Fahrenheit's scale between the temperatures of freezing and boiling water, is to that of Reaumur's scale as 9 to 4. Therefore if a given number of degrees of Fahrenheit's scale be multiplied by 4, and divided by 9, the result will be the corresponding number of degrees of Reaumur's scale.

higher

higher degrees of heat is important both to the philosopher and to the artist. To supply this deficiency * Mr. Wedgwood proposed, that an instrument should be formed of clay, and that the higher degrees of heat should be measured by its contractions, which render it an apparent exception from the general observation that all bodies are expanded by heat. He found that the diminution takes place in a low red heat; and that it proceeds regularly as the heat increases, until the clay becomes vitrified, and consequently to the utmost degree which crucibles or other vessels made of this material can support. The total contraction of some good clays, which he examined in the strongest of his own fires, was considerably more than one fourth part in every dimension. He proposed that the clay should be carefully formed into small pieces with two parallel sides at the interval of five tenths of an inch; and that a gage should be made of two pieces of brass, twenty-four inches long, fixed five tenths of an inch distant at the one end, and three tenths at the other, upon a brass plate, so that one of the thermometers should just fit at the wider end. If it be diminished in the fire one fifth of its bulk, it will then pass on to half of the length of the gage; if diminished two fifths, it will go on to the narrowest end; and in any intermediate degree of contraction the degree at which it stops will be the measure of its contraction, and consequently of the degree of heat which it has undergone. The scale of this thermometer begins at a red heat fully visible in day-light; and the greatest heat

* Phil. Trans. vol. 72. part 2. and vol. 74. part 2.

which

which Mr. Wedgwood obtained in his experiments was of 160 degrees, the gage being divided into 240 equal parts. We need not be solicitous about the precise degree of heat employed in baking these thermometers, provided only that it does not exceed the lowest degree, which we shall have occasion to measure; for a piece, which has suffered any inferior degrees of heat, answers as well for measuring higher degrees, as a piece which has never been exposed to the action of fire. * It will not however begin to suffer a diminution of bulk, until it shall have been heated beyond that degree of heat which it had before experienced. The only precaution necessary in the adoption of this thermometer, is that the same kind of clay should always be used.

* This quality of clay was curiously applied by Mr. Wedgwood to the discovery of the heat, by which many of the porcelains and earthen-ware of distant nations and different ages had been fired. By the thermometer he marked the heat at which they began to shrink, and consequently the greatest heat which they had before experienced. Of several pieces of ancient Roman and Etruscan wares, none appeared to have undergone a greater heat than 32, and none less than 20 degrees. Worcester china vitrified at 94; Mr. Sprimont's Chelsea china at 105; the Derby at 112; and Bow at 121; but Bristol china shewed no appearance of vitrification at 135. The common sort of Chinese porcelain did not perfectly vitrify by any fire which Mr. Wedgwood could produce; but began to soften about 120, and at 156 became so soft as to sink down, and apply itself close upon a very irregular surface beneath. The true stone nankeen, by this strong heat, did not soften in the least. The Dresden porcelain is more refractory than the common Chinese, but not equally so with the stone Nankeen. The cream-coloured, or Queen's ware, bears the same heat as the Dresden.

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Mr. Wedgwood afterwards contrived to connect his scale with that of Fahrenheit, by the use of a silver expansion-piece as an intermediate measure. A gage is formed for this in the same manner as for the thermometer of clay. The point near the narrow end of the gage, at which the piece stood, when of the temperature of 50 degrees of Fahrenheit's scale, was marked 0. By * observing the progress of the piece during the successive augmentations of its temperature, first to that of boiling water, and then to that of boiling mercury, it was discovered that one division of the gage corresponded to 20 of Fahrenheit's thermometer. The expansions of the silver were then compared with the contractions of the clay in higher temperatures; and by combining both results it was discovered that one degree of Mr. Wedgwood's thermometer corresponded to 130 of that of Fahrenheit, and that the 0 of the former corresponded to $1077\frac{1}{2}$ degrees of the latter. The whole scale of Mr. Wedgwood's thermometer includes 31,200 degrees of Fahrenheit's scale, or nearly 54 times as many as are in the latter interposed between the † freezing and boiling points of mercury.

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* The gage for the expansion-piece must be heated to the same temperature with the silver; because the latter would otherwise lose some of its expansion. This is not necessary in regard to the clay, because the contractions caused by heat are permanent. The gage being heated, the observation will give only the comparative expansion of the silver; but this will equally supply a graduated measure of temperature.

† Mercury freezes at 40 degrees, or rather at 39, below 0 of Fahrenheit's scale; and consequently 71 or 72 degrees below the temperature.

The grand discovery of the absorption and evolution of heat was made about the same time both by M. De Luc and Dr. Black; but the latter established it by the following experiments.

It was first ascertained, that ice in melting absorbs without change of temperature a considerable quantity of heat. This he proved by exposing in Florentine flasks to the same heat two equal bulks of water and ice both at the temperature of 32 degrees. The temperature of the water was raised 7 degrees in half an hour; but that of the melting ice continued unvaried ten hours. Hence he inferred, that during the liquefaction of the ice as much heat was absorbed, as is equivalent to 7 degrees multiplied by

20 or

the quantity of heat required to raise the temperature of freezing water. This was ascertained in the year 1781 by Mr. Hutchins at Hudson's Bay. The farther contractions of mercury take place in the act of freezing, and do not measure temperature. M. Braun, Professor of Philosophy, first congealed mercury at Petersburg, in December 1759, by artificial cold; but it has been since frequently observed in a state of congelation occasioned merely by the natural cold of the atmosphere. The latter has been remarked in Siberia, the northern countries of the continent of Europe, and at Hudson's Bay. All however that we are authorized to conclude in regard to the climate even of Siberia, is that it not unfrequently is of a temperature more than 39 degrees lower than 0 of Fahrenheit's scale. This is indeed sufficiently severe. When the thermometer is at 40 degrees below 0, the beams of houses crack with a loud explosion, trees split and are killed, birds fall down dead out of the air, and it is with the utmost difficulty that man, notwithstanding all his resources, can preserve the extreme parts of his body from being destroyed by the frost. It must however be remarked, that we have not any proof, that the difference of climates amounts to so much as the

variation

20 or the number of half hours, that is to 140 degrees. That the heat was really absorbed appeared from two considerations: 1. that the temperature of ice cooled below the freezing point, or 32 degrees, is raised by the application of heat, which indicates that ice is capable of receiving heat; and 2. that a thermometer applied at the bottom of the flask fell, which indicated the descent of the air cooled by the separation of part of its heat.

The heat thus absorbed Dr. Black denominated *latent* heat, because it did not affect the thermometer.

That heat is on the contrary evolved in congelation, may thus be proved. Water, when perfectly at rest, will fall several degrees below the freezing point without ceasing to be fluid. If it be then disturbed, the congelation will begin, and the temperature will at the same time be raised to 32 degrees.

As heat is absorbed in the change of aggregation from solidity to fluidity, so is it likewise in the change from fluidity to vapour. To prove this water was heated from 54 degrees to 212, or the boiling point, in 4 minutes; and

variation between one winter and another. The point of congelation of mercury is, on account of the sudden contraction of freezing mercury, best ascertained by a spirit-thermometer. By such an instrument Mr. Hutchins ascertained, that the greatest artificial cold, which could be produced even with the assistance of such natural cold as froze mercury, was only such as would correspond to 45 or 46 degrees below 0 of a mercurial thermometer. Dr. Blagden's history of the congelation of Mercury. Phil. Trans. vol. 73. part 2.

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consequently

consequently had received in that time 158 degrees of heat. It was intirely dissipated in 18 minutes more, and must therefore have received 869 degrees before it was all evaporated. The heat of the steam was however found to be exactly 212, as was also that of the boiling water. A great quantity of heat therefore must have entered into the composition of the steam in a latent state.

That heat is evolved in the condensation of vapour appears from the heat communicated to the water in the refrigeratory of a still. This water is applied to the tube through which the vapour passes, for the purpose of condensing it into a fluid; and the temperature which it acquires must be the effect of the heat evolved from the vapour in its change of aggregation to a fluid state.

These experiments enable us to explain the stationary temperatures of water whilst freezing or boiling. When water begins to freeze, it consequently begins to evolve heat, which supplies the same quantity of temperature, that is withdrawn by the refrigerating power of the surrounding medium. The water will therefore continue at the temperature of 32 degrees during the whole time of this evolution of heat, that is during the whole progress of the congelation. In the like manner when water begins to boil, the steam absorbs heat, and thereby carries away whatever is supplied beyond the temperature requisite for its own production.

The evolution and absorption of heat, by retarding the vicissitudes of heat and cold, are productive of much

much advantage. If these did not happen, the whole of any quantity of water which was exposed to a freezing temperature, would speedily become solid; and, on the contrary, a body of ice, exposed to a temperature higher than 32 degrees, would as speedily become fluid; and the changes of temperature would be uninterrupted. But, by this law, the temperatures which produce these effects, are for some time counteracted; and the constitutions of animals and vegetables are probably thereby enabled to sustain the vicissitudes of seasons. It should also be remarked, that in cold countries the sudden dissolution of large quantities of ice would occasion great annual inundations, which are prevented by the retardation, that is the consequence of this law. It is also serviceable in bringing nearer to equality the temperatures of different seasons and climates. "Thus," * says Dr. Crawford, "the diurnal heats are moderated by the evaporation of the waters on the earth's surface, a portion of the fire derived from the sun being absorbed and extinguished by the vapours at the moment of their ascent. On the approach of night the vapours are again condensed, and falling in the form of dew, communicate to the air and to the earth, the fire which they had imbibed during the day. In the regions near the poles, when the vernal and summer heats prevail, provision is made for tempering the severity of the winter colds; a quantity of elementary fire, upon the dissolution of the ice and snow, being absorbed by the waters, and deposited as it were in a great magazine, for the purpose of

* Treatise on Animal Heat. 2nd. Ed. p. 432.

mitigating the intensity of the cold when the frost returns—A considerable portion of the heat, which is excited by the action of the solar rays upon the earth's surface, within the tropics, is absorbed by the aqueous vapours, which being collected in the form of clouds, are spread like a canopy over the horizon, to defend the subjacent regions from the direct rays of the sun. A great quantity of elementary fire is thus rendered insensible in the torrid zone, and is carried by the dispersion of the vapours to the north and to the south, where it is gradually communicated to the earth when the vapours are condensed."

By these experiments we are also enabled to explain the effect of frigorific mixtures, as that of salt and snow. The substances mixed become fluid, and the fluid mixture absorbing much heat, deprives the neighbouring bodies of a part of that heat which they had before retained. * This effect however cannot happen, if the neighbouring bodies have already the same temperature to which the mixture could have reduced them; and at a lower temperature the mixture, by losing a part of its own heat, will become solid.

* Dr. Priestley's Heads of Lectures on Experimental Philosophy, lect. 30. Cold is in the same manner produced by evaporation. Thus ice is formed in the East Indies by exposing water to a serene air, at the coldest season of the year, in shallow porous earthen vessels. Part of the water transudes through the vessel, and evaporates from the outside; whilst the warmth of the earth is intercepted by bodies which are bad conductors of heat. Phil. Trans. vol. 65,

Since

Since a body in its change of aggregation from solidity to fluidity absorbs heat without any augmentation of temperature, its capacity for heat is said to be increased; and for a similar reason in its farther change from fluidity to vapour its capacity is again said to be increased. In the contrary changes, on account of the evolution of heat, its capacity is said to be diminished*.

The phenomena which have been described, seem

* Attempts have been made to determine at what degree in the scale of a thermometer a body would be deprived of all heat. For this purpose the following theorem was proposed by Dr. Irvine. Let s represent the required number of degrees of temperature of a body just congealed, reckoned from the total privation of heat, l the number of degrees of additional temperature required for reducing it to fluidity, n the capacity of the solid, and m the capacity of the fluid. Then, since the quantities of heat contained at the same temperature must be proportional to the capacities for heat, $s + l : s :: m : n$.

Whence $s = \frac{ln}{m-n}$. But, on account of the change of capacity which the body undergoes in becoming solid, this conclusion must be corrected in the reciprocal proportion of that change; for the changes of temperature of a body must be increased in the same proportion, in which its capacity is diminished. Therefore $n : m ::$

$\frac{ln}{m-n} : \frac{lm}{m-n} = s$ the number of degrees of temperature from the total privation of heat. It has been observed that water in becoming ice evolves one tenth part of its heat, so that $m : n :: 10 : 9$. If

then l be considered as 140, $\frac{lm}{m-n} = \frac{140 \times 10}{1} = 1400$. If therefore ice were cooled 1400 degrees below 32, that is 1368 degrees below 0, it would retain no more heat. Dr. Crawford, from a consideration of the heat of pure and inflammable air, determined the point of total privation to be nearly 1500 degrees below 0. *Treatise on Animal Heat.* 2nd. Ed. p. 267.

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to warrant the opinion, that heat is a fluid, which, by combining in various degrees with the particles of bodies, produces the changes of aggregation which they experience, and the effects that have been noticed under the names of absorption and evolution. The increase of temperature occasioned by friction or percussion has been explained agreeably to this hypothesis by Mr. Kirwan. If the parts of a body containing any fluid be made to vibrate strongly and irregularly, they will expel a part of the fluid out of their pores, provided that the fluid be not sufficiently compressed to move in correspondence with the vibrations. For in this case a vibrating particle may be considered as if its dimensions were increased, which is in effect the same thing as if the pores of the body were diminished. The capacity of the body will thus be diminished, and consequently its temperature will be increased*.

But,

* Count Rumford has indeed lately contended (*Journal of Nat. Philosophy &c.* for June, 1798) for the ancient opinion, that heat is only a motion excited in the particles of bodies. This he has been induced to do, by observing "the very considerable heat which a brass gun acquires in a short time in being bored, and the more intense heat (much greater than that of boiling water, as he found by experiment) of the metallic chips separated from it by the borer." This heat, he argues, could not be furnished by the metallic chips, because it appeared from experiment, that their capacity for heat was not altered. It was next to be considered, whether the heat might not have been supplied by the adjacent air. To determine this question, he excluded the external air from all access to the inside of the bore, and even to the insertion of the borer, and found the result of the experiment still the same. Lastly, he argues that it is utterly improbable, that the heat could have been supplied by means of the iron bar, to the end of which

But, whatever may be the nature of heat, it must be regarded as holding a principal place in the series of causes, by which the adjustment of the universe is maintained. As the projectile force, combined with the attraction of Gravity, regulates the motions of the great bodies which compose the planetary system; so it is the repulsive force of heat, combined with Corpuscular Attraction, that regulates the various changes of the smaller particles of matter: and as the different projectile forces are adapted to the different planetary movements, so the unequal capacities of different substances for heat maintain them in different states of aggregation, though their circumstances are in other respects similar.

The attraction, which has been hitherto considered, is that of aggregation, or that which exists between principles of the same nature. The attraction of composition, which retains two or more principles of different

which the steel borer was fixed, or of the small neck of gun-metal, by which the hollow cylinder was turned round the borer. On this last consideration the determination of the cogency of the Count's conclusion seems to depend. He maintains the improbability of this supply, because "heat was continually going off or out of the machinery by both these passages, during the whole time the experiment lasted." This however, as has been suggested by Dr. Perceval, is reducible to this general query: may not heat, which had been latent in one body, be communicated from it by means of another, which, on account of its inferior capacity for heat, is at the same time evolving a part of it in a sensible state? The rotation of the neck which turns the cylinder, and the vibrations excited in the iron bar to which the borer is affixed, must cause agitations in the air, and thereby extricate heat; and the supply must be unlimited.

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natures in a state of combination, must now be considered.

* If two solid bodies, disposed to unite together, be brought into contact with each other, the particles which touch will combine, and form a compound; and if the temperature at which this new compound assumes the fluid form, be higher than the temperature of the experiment, the process will go no farther, because this new compound, being interposed between the two bodies, will prevent their further access to each other: but if, on the contrary, the freezing point of the compound be lower than this temperature, liquefaction will ensue; and the fluid particles being at liberty to arrange themselves according to the law of their attractions, the process will go on, and the whole mass will gradually be converted into a new compound in the fluid state. An instance of this may be exhibited by mixing together common salt and perfectly dry pounded ice. The salt alone will not liquefy unless very much heated, nor the ice at a lower temperature than 32 degrees of Fahrenheit's thermometer. But brine, or the compound of these two, will become liquid at any temperature which is not more than 38 degrees below the freezing point of water. If then the temperature of the experiment be within these 38 degrees, the first combinations of salt and ice will produce a fluid brine, and the combination will proceed until the temperature of the mass has gradually sunk as low as the freezing point of brine; after which

* Nicholson's Dictionary of Chemistry. Art. Attraction.

it would cease, if it were not that surrounding bodies continually tend to raise the temperature. And, accordingly, if the ice and salt be previously cooled below the temperature of freezing brine, the combination and liquefaction will not take place.

The instances, in which solid bodies thus combine together, not being very numerous, and the fluidity ensuing immediately after the commencement of this kind of experiment, have induced several chemists to consider fluidity in one or both of bodies applied to each other, to be a necessary circumstance, in order that they may produce chemical action upon each other.

If one of the two bodies applied to each other be fluid at the temperature of the experiment, its parts will successively unite with the parts of the solid, which will thus be suspended in the fluid, and disappear. Such a fluid is called a solvent or menstruum, and the solid body is said to be dissolved.

Some substances unite together in all proportions. In this way the acids unite with water. But there are likewise many substances which cannot be dissolved in a fluid at a settled temperature, in any quantity beyond a certain proportion. Thus water will dissolve only about one fourth of its weight of common salt; and if more be added, it will remain solid. A fluid which holds in solution as much of any substance as it can dissolve, is said to be saturated with it. But saturation with one substance is so far from preventing a fluid from dissolving

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another body, that it very frequently happens that the solvent power of the compound exceeds that of the original fluid itself.

Heat in general increases the solvent power of fluids, probably by preventing part of the dissolved substance from congealing, or assuming the solid form.

It often happens, that bodies which have no tendency to unite are made to combine together by means of a third, which is then called the medium. Thus water and fat oils are made to unite by the medium of an * alkali in the combination called soap.

It very frequently happens, on the contrary, that the tendency of two bodies to unite, or remain in combination, is weakened or destroyed by the addition of a third. Thus, spirit of wine unites with water in such a manner as to separate most salts from it.

From a great number of facts of this nature it is clearly ascertained that some bodies have a stronger tendency to unite than others; and that the union of any substance with another will separate a third substance, which had been previously united with one of them, except only in those cases wherein the new compound has a tendency to unite with that third substance, and form a triple compound. This preference of uniting, which a given

* Alkalis are salts of a caustic taste, whose most distinguishing property is that they change blue vegetable colours to green.

substance

substance is found to exhibit with regard to other bodies, is by an easy metaphor called *Elective Attraction*; and is divided into two cases, those of *single and double elective attractions*.

When a substance is applied to another substance compounded of two ingredients, and unites with one of these, so as to separate the other, this effect is said to be produced by *single elective attraction*. Thus, an alkali added to a solution of Epsom salt, which is a combination of magnesia and the sulphuric acid, separates the magnesia from this compound, and unites itself with the acid. The magnesia, being insoluble alone, falls to the bottom in the form of powder; and the alkali, uniting with the acid, forms a soluble salt, and is suspended in solution. The magnesia is in this case called a *precipitate*. So likewise, if a combination of lead and sulphur be fused with iron, the sulphur unites with the iron, and leaves the lead free.

† The precipitate from a solution may be formed in three different ways. 1st. One of the two principles, which formed the original combination, may be the precipitate; as in the separation of magnesia from the Epsom salt. 2^d. The new compound itself, formed by the combination of the third principle with one of those which composed the original combination, may be the precipitate. Thus, when the sulphuric acid is poured

* Acids are salts of a sour taste, whose most distinguishing property is that they change blue vegetable colours to red.

† Chaptal's Elements of Chemistry: part 1 sect. 1]

on a solution of lime in the marine acid, the marine acid is disengaged in vapours, and the sulphuric acid forms with the lime a solid precipitate. 3ly. The disengaged body and the new compound may be precipitated together. Thus, when Epſom ſalt is diſſolved in water, and lime-water is added, the magnesia is precipitated from the original compound, and the new compound of lime and the ſulphuric acid is alſo precipitated as in the 2nd. caſe.

Double elective attraction takes place, when two ſubſtances being applied to each other, mutually exchange an ingredient, and two new compounds of properties different from thoſe of the former are produced. Thus, if marine ſalt of lime, which is compoſed of marine acid and lime, be applied to mild volatile alkali, which is compoſed of volatile alkali and fixed air, the volatile alkali of the latter compound will combine with the marine acid of the former to form ſal ammoniac, and the fixed air and lime will compoſe chalk. In conſequence therefore of this proceſs ſal ammoniac and chalk are produced inſtead of marine ſalt of lime and mild volatile alkali. In order that this interchange ſhould happen, it is neceſſary that the ſum of the attractions which exiſt between the ingredients of the one compound and thoſe of the other, ſhould be greater than the ſum of the attractions which tend to retain them in their original combinations.

Many circumſtances modify the effects of elective attractions.

tractions. 1st The * temperature, acting differently upon the several parts of compounded bodies, seldom fails to alter, and frequently reverses, the effects of the elective attractions. Thus, if spirit of wine be added to the solution of nitre at a common temperature, it will unite with the water, and precipitate the nitre; but if the temperature be raised, the spirit will rise on account of its volatility, and the nitre will be again dissolved. 2^{ly}. The comparative quantities of the substances will vary the results. If to a compound of two substances a third be added beyond a certain proportion, the redundant quantity may again dissolve the precipitate. Thus, if an alkali be added to a metallic solution in a certain proportion, the metal will be precipitated; but if a greater quantity of the alkali be added, the precipitate will be

* Count Rumford thinks it not improbable, that every change of state, in every kind of substance, may be owing to heat alone; that every concretion is a true congelation effected by cold, or the diminution of heat; that every change from a solid to a fluid form is a real fusion;—and that it may be found, that the apparent violence, with which certain solids are attacked by their solvents, is not owing to any particular or elective attraction, but to the considerable degree of heat or cold, and the great difference of comparative weight which ensues in the solvent, from this cause as well as from the subsequent change produced by combination.—An instance of these two causes operating jointly is adduced in the solution of common salt in water. If this solid be supported in a perforated vessel under water, but near its surface, the solution will be most rapid: 1st because the temperature is diminished, and consequently the water is condensed in the process; and 2^{ly}. because the solution of salt in water is itself more dense than the water itself. On both accounts therefore the brine will rapidly descend, and fresh portions of the solvent will continually be brought in contact with the salt. *Experimental Essays.*

again

again dissolved. If on the other hand, one of the ingredients of the compound body be * in excess, the addition of the third substance may combine with that excess, and leave a substance exhibiting properties very different from the former. Thus, if cream of tartar, which consists of the vegetable alkali united to an excess of the acid of tartar, be dissolved in water, and chalk be added, the excess unites with a part of the lime of the chalk; and forms a scarcely soluble salt; and the compound formed of the remaining acid and the vegetable alkali is a salt, greatly differing in taste and other properties from the cream of tartar. 3ly. The solubility or insolubility of substances at the temperature of the experiment must considerably affect the results. It is evident, that many separations may ensue without precipitation; because this does not happen unless the separated substances be altogether, or nearly, insoluble.

The operation, by which iron is said to be changed into copper, is only a case of elective attractions. Bars of iron are laid in water, which springs from a copper mine, and is impregnated with particles of copper dissolved in a mineral acid. This acid, having a stronger elective attraction towards iron than towards copper, will, when the iron is presented to it, dissolve a portion of it,

* There is a certain proportion of the ingredients, in which the properties of the compound are most remote from those of its ingredients. When either of them exceeds this proportion, so as to predominate in the compound, it is said to be in excess. In the former case the compound is said to be a neutral one.

and precipitate the copper in its place. The bars are accordingly found incrusted with copper.

This inequality of attractions is the principle of all the decompositions effected either by nature or art. If all bodies had the same affinity, no change could take place among them; but, because their attractive forces are unequal, the ingredients of any compound may be separated in various ways by the application of other substances.

In the change of aggregation from fluidity to solidity, the particles, which had been suspended either by solution or fusion in the fluid state, continually tend to compose bodies of symmetrical forms. On account of the resemblance which these conformations bear to that of rock-crystals, the process has been denominated Crystallization. That the form of a crystal may be perfect, three circumstances are required; time, a sufficient space, and rest. If the operation be not performed slowly, the particles cannot approach each other in that gradual manner, which is necessary for their regular arrangement; if the space be confined, their approach must be influenced by the constraint of their situation; and any agitation will necessarily disturb the symmetry of their combination. This process is most remarkable in solutions of salts; and it has been observed, that different salts assume different figures. Since heat generally increases the solvent power of fluids, they must by cooling suffer a part to precipitate; and for this reason crystallization always begins at the surface of the liquid, and on the sides of the
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the containing vessel, because these parts first suffer refrigeration. When * water freezes slowly, it forms regular crystals. They are long, and joined together in such a manner, that the smaller are inserted into the sides of the greater; and thus these compound crystals have the appearance of feathers, or of branches of trees with leaves. The angle formed by the insertion of the smaller crystals into the longer is of 60 degrees. The crystals of metals may be seen by fusing them in a vessel having a hole in the bottom, which is stopped with a stopper; for, if the surface be suffered to congeal, and the fluid metal be suffered to run out through the hole, the inferi-

* The tendency to crystallize, which may perhaps begin 8 degrees above the freezing point, is probably, as Dr. Percival has observed, the reason why water affords an apparent exception from the general observation that all bodies are condensed by cold. The particles of water may really be brought nearer to each other in each distinct crystal, and yet, on account of the reticulated disposition of the crystals, the whole mass may be increased in its dimensions.

Count Rumford has remarked, that this peculiarity is productive of the most important advantage in the preservation of heat. He has shewn that heat is communicated in a fluid by the motions of its particles, which are probably caused by the inequality of their expansions. But, on account of this peculiarity of water, these motions are stopped by a contrary effect before the water has been cooled down to the freezing temperature; and consequently much of its heat is retained. This circumstance therefore preserves the fluidity of lakes of fresh water, and of the sap in vegetables. The sea-water on the other hand, holding a large portion of salt in solution, is not subject to this law, and will therefore continue to communicate heat to the atmosphere; and this communication of heat will be the more copious, because its point of congelation is at a considerably lower temperature. *Experimental Essays.*

or surface of the remaining metal will be seen * crystallized. The crystals of mercury are formed by freezing, and are much more conspicuous than those of any other metal. Dr. Blagden conjectures, that the cause of this distinction is probably that it congeals more slowly than any other metal, as probably it has never been exposed to a degree of cold much below its freezing point. The crystals of earths and stones are formed in many cases by the depositions of water, when it has by attrition reduced them to a state nearly correspondent to solution. All gems are specimens of crystallized earths or stones, and basaltes are considered as of the same class.

The earlier mineralogists and chemists endeavoured to arrange bodies by the figures of their crystals. Subsequent experience has however shewn, that their crystallization is variable by so many and such minute circumstances, that a considerable dependance on this single attribute is very liable to error.

A very singular property may be observed in solutions of salts, which has some connection with crystallization, though it has a dependance on the operation of other causes. This is called Saline Vegetation. The saline matter slowly creeps up the sides of the vessels in which it is contained, passes over the edge, and down the ex-

* The permanent texture of bodies appears to depend upon the state of crystallization. Thus steel, suddenly cooled, breaks with a granular fracture, is less dense, and very hard; whereas the same steel, more slowly cooled, is denser, softer, less brittle, and when broken exhibits a very different internal texture. Nicholson's Dictionary of Chemistry. Art. Crystallization.

ternal surface. This seems to arise from the combined operation of light and air. * M. Chaptal, in a variety of experiments, ascertained that the vegetation took place only on those surfaces which were illuminated. In the space of a few days, and frequently even within one single day, the salt was elevated several lines above the liquor upon the enlightened surface, while there did not appear the smallest crust or edge on the dark part. This was most remarkable in the metallic salts which are combinations of acids with metals. The vegetation may be directed at pleasure towards the different parts of the vessel, by covering them in succession; for the vegetation always takes place in the enlightened part, and quickly ceases in that which is covered. When the same solution has stood for several days, the insensible evaporation gradually depresses its surface, and a crust or edge of salt is left in the obscure part; but the salt never rises, or at least very imperfectly, above the liquor, and cannot be compared with the true vegetation.

This property of vegetation differs greatly in the several salts. Those which are deliquescent, that is, which are reduced to fluidity by the moisture which they attract from the air, moisten the sides of the vessel to a small distance above the edge of the fluid, but form neither crust nor ramification. Those salts which are least deliquescent, appeared in general to vegetate most speedily; and among these the metallic salts appeared to have the pre-eminence. Very singular varieties are observable in the forms affected by the several salts in their vegeta-

* Nicholson's Dictionary of Chemistry, Art. Vegetation, Saline.

tion. The vegetation of the salt of tin appeared to take root in the liquor by a multitude of oblong pyramidal crystals which entered into the solution.

M. Chaptal was however convinced that air is the principal agent; by observing 1. that a solution will not vegetate, though exposed to light, if it be covered by a glass from the air; 2. that it vegetates sooner in a very open than in a cylindrical vessel, and sooner in the latter than in an uncorked bottle; 3. if a glass funnel be reversed in a vessel containing a saline solution, the vegetation will take place on the external surface, but scarcely at all within; 4. a solution of copperas, left in a very obscure place, vegetated in that part only which was uncovered, but more slowly than when a greater quantity of light was admitted. It appears therefore that the access of air and its free circulation are peculiarly advantageous to the production of this appearance.

"The presence of light," Mr. Nicholson adds, "is found to interrupt crystallization nearly in the same manner as agitation would have done. The vegetation of salts consequently appears to be of a distinct nature in certain respects from crystallization within a fluid. The crystallization which approaches most nearly to vegetation is effected when a thin covering of saline solution is spread out upon a pane of glass. In this case the attractions of the particles of the salts to each other, are all nearly in the same plane, and may therefore, notwithstanding the speedy evaporation, be expected to produce effects more symmetrical than when a much greater

thickness of fluid is said to crystallize. The manner in which the air acts is a subject of no difficulty; since it favours the crystallization by abstracting the water. But the agency of light in the experiments of M. Chaptal is much more obscure. Experiment leads us no farther than to assert, that it singularly favours the assumption of the elastic state, insomuch that principles, which in the dark would have remained united, become separated by the agency of light which gives elasticity to one or more of them.—Simple evaporation is likewise so far favoured or modified, that the fluid in a closed vessel or bottle partly filled with water or saline solution, rises and is condensed in drops on the side nearest the light. This fact helps us forward in a certain degree: for the light must raise part of the solution, whether merely aqueous or saline, on that side of the vessel which is most illuminated; and when once the surface is wetted, the saline solution will rise to a certain height by cohesive attraction. In this situation it becomes the film of liquid exposed on a pane of glass. Speedy evaporation affords crystals more or less regular, on the same principle as ramified crystals are produced in the instance last mentioned. The interstices between these minute crystals are capillary tubes, which carry up more of the saline solution, which is distributed by the agency of light as before, and the vegetation goes on. Hence it appears, that a want either of light or of air must suspend the process, since it is in vain that the light is found to spread the fluid over the surface in a closed vessel, if there be not enough of air in succession to crystallize the salt by evaporation.” It must be observed, that the nature of the
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the metals is not a matter of indifference in these experiments. Glass is very proper, but metals much less adapted. The cause of this difference has not been ascertained.

Since all bodies are capable of acquiring the vaporous state, in which their particles repel each other, there seems to be a Corpuscular Repulsion as well as Attraction. M. Chaptal however does not think that there is in this case any real repulsion. * According to his opinion this effect is produced, partly by the endeavour of the matter of heat to combine with bodies, which necessarily diminishes their force of aggregation; and partly by the extreme levity of that matter, which continually tends to elevate the body with which it is combined. But whether this be a real repulsion or not, chemistry has in this case brought within our knowledge one of those admirable contrivances, by which the equilibrium of the material world is preserved. "The affinity of composition," says M. Chaptal, "is so much the more strong as bodies approach nearer to the elementary state; and we shall observe, on this subject, that this law of nature is founded in wisdom: for if the force or affinity of composition did not increase in proportion as bodies were brought to this degree of simplicity; if bodies did not assume a decided tendency to unite and combine, in proportion as they approach to their primitive or elementary state; the mass of elements would continually increase by these successive and uninterrupted decomposition-

* Elements of Chemistry, part 1. sect. 1.

ons; and we should insensibly return again to that chaos or confusion of principles, which is supposed to have been the original state of the globe." The emission and reflection of the rays of light, and the phenomena of Electricity and Magnetism furnish less equivocal instances of a repulsive power; and it has been observed, that the convex glass of a telescope will lie on another glass of the same kind without contact, since colours are reflected every where from between them, which could not happen if the two in any point united to compose one uniform medium of glass. The floating of small portions of an heavy substance, as small needles, on the surface of a lighter fluid, is also considered as an effect of a repulsive force, which prevails between the floating bodies and the fluid until they have been brought into close contact; for these bodies, when wetted, descend to the bottom.

C H A P. III.

OF ELECTRICITY.

OUR knowledge of the forces denominated Electricity and Magnetism is yet so imperfect, that it may be doubted whether the consideration of them should be included in an elementary treatise; but their great importance seems to require, that such properties of each as have been clearly ascertained, should not be omitted. The modern discoveries concerning the former have enabled us to guard against the ravages of lightning, and promise us a more intimate acquaintance with the most curious operations of nature; and to the properties of the latter must be ascribed the extension of modern navigation, and the consequent enlargement of human intercourse, and improvement of the blessings of civilized society.

The substance of this chapter has been taken from Cavallo's Complete Treatise on Electricity, 4th Ed.

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The word Electricity is derived from the Greek name of amber, in which the property of attracting light bodies was remarked by Thales, who flourished about 600 years before the Christian æra. Theophrastus, who flourished 300 years after Thales, mentioned as possessing the same property another substance, which is supposed to have been that now called tourmalin. From this time to that of Gilbert, whose treatise on the Magnet was published in the year 1600, no person appears to have given any attention to this part of philosophy. He observed that the property of attracting light bodies after rubbing was possessed by many other substances. But the discovery which was accidentally made in the year 1745, of the great accumulation of this power in what is commonly called the Leyden phial, rendered Electricity the object of general curiosity.

If a person, holding with one of his hands a clean and dry glass tube, rub it with his other hand, also clean and dry, stroking it alternately upwards and downwards, and after a few strokes present to it small light bits of paper, thread, metal, or any other substance, the rubbed tube will immediately attract them, and after a little time repel them; then attract them again, and so alternately for a considerable time. If the tube be rubbed in the dark, and a finger be then presented to it at the distance of about half an inch, a lucid spark will be seen between the finger and the tube, a snapping noise will be heard, and the finger will receive a push, as if from air issuing with violence out of a small tube. The
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cause of these phenomena is named Electricity; and all substances which are capable of producing them by any means are called Electrics.

If at the end of the tube remote from the hand be tied a wire suspending a metallic ball, and the tube be excited by rubbing as before, the metallic ball will exhibit all the properties of the excited tube, the electric power passing through the wire to the ball: but if, instead of the wire, a silk string be used, the ball will not shew any signs of Electricity, the silk string not permitting the electric power to pass from the tube to the ball. The wire is therefore called a Conductor, and the silk string a Non-conductor of Electricity. A body resting intirely upon non-conductors, as the ball when suspended by the silk string, is said to be Insulated.

The first law of Electricity is that all known bodies are in regard to it reducible to the two classes of Electrics and Conductors.

Experiments shew, that whatever substance may be excited, will not conduct Electricity; and, on the contrary, that whatever substance will conduct this power, cannot be excited. These principles must however be received with two qualifications. 1st There is not any known body, which is either a perfect electric, or a perfect conductor; the electric power being in part transmitted through, or over the surface of most, and perhaps all the electrics, and finding some resistance in going through the best conductors. 2^{ly} The difference between electrics and con-

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ductors

ductors is connected with the * temperatures of bodies. Glass and other electrics become conductors when heated; and, on the contrary, ice, cooled to 13 degrees below 0 of Fahrenheit's scale, becomes electric.

The two following lists contain, in general, all the electrics and conductors, disposed in the order of their perfection.

Electrics.	Conductors.
Glass, and all vitrifications.	Gold.
Precious stones; of which the most transparent are the best.	Silver.
Resins, and resinous compounds.	Copper.
Amber.	Brass.
Sulphur.	Iron.
Baked wood.	Tin.
Bituminous substances.	Quicksilver.
Wax.	Lead.
Silk.	Semi-metals.
	Ores; of which the best are those which contain the greatest quantity of

* Mr. Nicholson hence concludes (Introduction to Nat. Phil. vol. 2. p. 303.) that there is reason for conjecturing, "that there is a certain degree of heat, at which a given body may be at the medium between perfect conducting and non-conducting, above which degree it becomes a conductor, and beneath a non-conductor." But the properties of the tourmalin, which have been found to belong also to several hard precious stones, will not suffer us to establish this general principle. These shew no signs of Electricity, whilst kept in the same degree of heat, but become electrical either by increasing or diminishing their heat.

Electrics.

Electrics.	Conductors
Cotton.	metal, and are nearest
Dry animal substances, as feathers, wool, hair, &c.	to a metallic state.
Paper.	Charcoal.
White sugar, and sugar-candy.	Animal fluids.
Air.	Other fluids, except air and oils.
Oils,	Effluvia of flaming bodies.
Calces of metals, and semi-metals.	Ice.
Ashes of animal and vegetable substances.	Snow.
Dry vegetable substances.	Most saline substances; of which the metallic salts are the best.
Hard stones; of which the hardest are the best.	Stony substances; of which the hardest are the worst.
	Smoke.
	Steam.

In these lists there are some ambiguous substances, which may be excited as electrics, and are notwithstanding tolerably good conductors. But the distribution of bodies into these two classes is so far accurate in this respect, that the less perfect conductor any substance is, the more nearly does it approach to the nature of an electric; and, on the contrary, the less perfect electric it is, the more nearly does it approach to the nature of a conductor.

Friction is the general method by which Electricity may be excited in all electric substances. Whether they be rubbed with electrics of a different kind. or with

conductors, they always shew signs of Electricity; but the Electricity is stronger, when the friction is performed with conductors. For obtaining a considerable Electricity, it is also necessary that the substance employed as a rubber should have a regular communication with the earth by good conductors. But there are other methods, besides friction, by which some electrics may be excited. These are pouring a melted electric into another substance, heating or cooling, and evaporation or effervescence. Electricity is excited by the first of these methods in sulphur, wax, and chocolate; by the second in the tourmalin, and several hard precious stones; and by the third in the evaporation of water from among ignited coals, in the simple combustion of coals, and in the effervescence of iron filings in diluted vitriolic acid.

For the more convenient excitation of Electricity various machines have been contrived, which are therefore called Electrical Machines. The most usual consists of an hollow cylinder of glass, which is made to revolve on its axis; a rubber of leather, which is applied to the surface of the glass by the pressure of springs; and a metallic conductor, which communicates with the cylinder. The Electricity excited in the cylinder is received by the conductor, and thereby more conveniently subjected to experiment. The steadiness of the conductor renders it more commodious for this purpose than the substance in which the Electricity is excited; but it derives also a considerable advantage from its conducting nature. The electric power, which it has received gradually from the electric, it can communicate at once; because the
Electricity

Electricity belonging to the whole conductor is easily transmitted through its substance to that side, at which the object designed for receiving it is presented. That the Electricity should not be speedily dissipated, it is necessary to insulate the conductor. It is therefore supported by a pillar of glass. The rubber is also insulated in the same manner, because a communication between the rubber and the earth may then be formed or removed at pleasure, by occasionally suspending from it or removing a wire or chain. A loose piece of silk is usually attached to the rubber, which adheres to the cylinder during its excitation, and, being a non-conductor, confines the Electricity. The excitation is assisted by rubbing on the rubber of the machine an amalgam, or mixture of one part of zinc and four or five parts of mercury.

When Electricity has been excited by such a machine, it is observed, that, if the rubber be insulated, effects of opposite kinds are produced by the cylinder and the rubber. 1st If a light body insulated, as a small piece of cork suspended by a thread of silk, be brought near to the conductor communicating with the cylinder, it will first be attracted, and then repelled; after which it will not again be attracted. The same will happen, if a light body be in the like manner brought near to another conductor communicating with the rubber. But if either of these, when repelled by the one conductor, be presented to the other, it will be attracted. 2^{ly} If two or more light insulated bodies be severally attracted by the conductor communicating with the cylinder, and, when afterwards repelled, be brought together, they will repel each

each other, and continue in this repulsive state for a considerable time. The same will happen, if other light bodies be in the like manner presented to the conductor communicating with the rubber. If then these two sets of light bodies be brought near to each other, they will, instead of repelling, attract each other; and every appearance of Electricity will cease. 3ly. If a pointed body, as a needle or wire, be presented in the dark to the conductor communicating with the cylinder, a lucid globule, resembling a star, will be seen upon the point; but, if it be presented to the conductor communicating with the rubber, a lucid pencil will appear, composed of rays seeming to issue from the point, and diverge towards the conductor,

These two Electricities are observable in many instances; since a rubber, whenever it is insulated, will exhibit the Electricity contrary to that of the excited electric; and almost all the electrics may be made to acquire at pleasure the one or the other Electricity, by using rubbers of different kinds. Even a difference in the nature of the surface of the same body will change the Electricity. Thus the excitation of smooth and of rough glass will produce opposite Electricities,

The Electricity derived in this experiment from the cylinder has been named the Vitreous Electricity, because it was supposed to be the constant production of rubbed glass; and the other, because it was first observed to be produced by resinous substances, was called the Resinous

nous Electricity. Since however * Dr. Franklin proposed the hypothesis of a single electric fluid, by whose excess or deficiency he endeavoured to explain the phenomena, the former has been usually termed the Positive, and the latter the Negative Electricity. According to this hypothesis all the phenomena called electrical are supposed to be effected by an invisible subtle fluid existing in all the bodies of the earth. It is supposed also, that the particles of this fluid repel each other, but attract those of all other matter. When a body does not shew any electrical appearance, it is then supposed to contain its natural quantity of electric fluid; but if it shew any Electrical appearance, it is supposed that it has either acquired an additional quantity, or lost some of its natural share. The phenomena of the preceding experiment are thus explained agreeably to this hypothesis. The Electricity, which is excited in the cylinder, and communicated to its conductor, is really by the friction drawn from the rubber and the bodies with which it communicates. This is confirmed by the consideration, that, if the rubber be insulated, little Electricity can be excited; the rubber in this case being able, according to the hypothesis, to supply the electric only with that small quantity of fluid which belongs to itself, or which it collects from the contiguous air. If therefore the cylinder become over-charged with Electricity, the rubber must be under-charged, and exhibit phenomena of an opposite kind.

The second law of Electricity is that bodies in oppo-

* Bishop Watson about the same time proposed the same theory.

like states of Electricity, and also an electrified and a non-electrified body, attract each other; and bodies in similar states of Electricity repel each other.

The truth of this law appears from the preceding experiment. The light bodies, whilst in a non-electrified state, are attracted by either conductor; and, when from the different conductors they have received opposite Electricities, they are observed to be attracted towards each other. On the contrary, the light bodies, which from either conductor have received similar Electricities, are observed to repel each other.

The case of attraction has been in some degree reconciled with the hypothesis proposed by Dr. Franklin; but that of repulsion has remained an acknowledged difficulty. Mr. Cavallo has indeed* laboured to establish the correspondence of this phenomenon with the theory. His solution however appears to be liable to objections. Perhaps the following method of explaining both cases may be thought more satisfactory.

For determining what should be the result in any combination of electrified bodies three forces must be considered; 1st the attraction subsisting between each body and its own portion of the electric fluid; 2^{ly} the attraction which may subsist between each body and the portion of the electric fluid belonging to the other; and 3^{ly} the

*Vol. 3. p. 192.

repulsion

repulsion subsisting between the two portions of the electric fluid belonging to the two bodies.

That the attraction subsisting between two bodies in opposite states of Electricity may be explained, it is necessary to consider previously the case of two bodies in their natural state. In this case the force subsisting between each body and its own portion of the electric fluid is not in a state of saturation, because it must be sufficiently strong to counterbalance the elasticity of the fluid. Each body is therefore still capable of being attracted by the fluid belonging to the other, and each portion of fluid is also capable of such attraction. This force, if it should operate alone, would draw the bodies together; but the mutual repulsion of the two portions of the fluid tends to produce the opposite effect. The quiescence of the bodies proves the equality of these forces.

If two bodies in opposite states of Electricity be brought together, the body positively electrified cannot be attracted by the electric fluid belonging to the other; because this body may be considered as saturated with the fluid, and that portion of the fluid may be considered as saturated with solid matter. For the opposite reasons an attraction will take place between the body negatively electrified and the fluid belonging to the former. It remains to be shewn, that this attractive force may exceed the mutual repulsion of the two portions of fluid. It must be observed, that the repulsion remains the same, because the sum of the two quantities of fluid is not altered; whereas the attraction is augmented by the unequal distribution

tribution of the fluid. The one body is charged with more fluid than that which its own attracting force is capable of retaining, and the redundant fluid will consequently be strongly impelled towards the other body, whose attractive power is at the same time increased by the deficiency of its own portion of fluid.

If one of the bodies be in its natural state and the other positively electrified, the repulsion is increased because the quantity of fluid is increased; but the attraction is still more increased, because the latter is charged with an additional quantity of fluid, which, as in the former case, is only retained by the non-conducting quality of the air. If the latter be negatively electrified, the repulsion is diminished, because the quantity of fluid is diminished; but the body in its natural state will be still more strongly attracted towards the electrified body, on account of the deficiency of fluid belonging to the latter, and this augmentation will compensate the cessation of the attractive force between the deficient fluid and the former body. In each case therefore the attraction will prevail. After that light bodies in their natural state have been attracted, they acquire by contact the same state of Electricity with that of the body by which they had been attracted, and are then repelled agreeably to the next case.

If two bodies be similarly electrified, they must repel each other. For, if they are both positively electrified, the solid bodies are saturated with the electric fluid; and, if they are both negatively electrified, the two portions of the electric fluid are saturated with the solid bodies.

Therefore,

Therefore in the former case each body must be incapable of attraction towards the fluid belonging to the other; and in the latter case the portion of fluid belonging to each body must be incapable of attraction towards the other body. Consequently in neither case will any force be exerted in opposition to that repelling force, which tends to separate the two portions of the fluid, and thereby to separate also the solid bodies with which they are connected*.

The third law of Electricity is that pointed conductors both discharge and receive *silently* the electric fluid.

This is a most important property, since it is by this that we are enabled to secure buildings from the violence

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of

* This theory supposes the electric fluid to be a *single* fluid, withdrawn from one body, and superadded to another; but it is not necessary to regard it as a *simple* fluid. We know that atmospheric air is a combination of at least two fluids; and yet explain all phenomena depending merely on its presence, as the suspension of quicksilver in the barometer, by considering it as one. Some other phenomena of the electric fluid may be connected with a decomposition of its nature. Hence possibly may be derived its phosphoric smell, the power which it is said to possess of changing blue vegetable colours to red, and its effect in increasing or diminishing the bulk of various kinds of air. But the theory of two electric fluids, to which the theory that has been stated is opposed, represents the phenomena of electric attraction and repulsion as arising from the alternate predominance of either of two fluids, each of which consists of particles repelling each other and attracting those of the other fluid and of all other bodies. In the natural state of a body these fluids are supposed to be restored to equilibrium by their combination. This theory

of lightning. It may be proved by the following experiment. Let a person hold the knob of a brass rod at a distance from the conductor of an electrifying machine, that sparks may easily fly from the latter to the former, when the machine is in motion. If then the sharp point of a needle be presented at a distance nearly double of the former, no more sparks will go to the rod: if the needle be wholly removed, the sparks will be seen again: if it be again presented, the sparks will disappear. These phenomena shew, that the point of the needle draws off silently almost all the fluid, which the cylinder of the machine throws upon the conductor. If the needle be fixed upon the conductor with the point turned outward, and the knob of the brass rod, or the knuckle of a finger, be brought very near to the conductor, though the excitation of the cylinder be very strong, no spark, or only an exceeding small one, can be obtained from the

theory seems to be refuted by an experiment contrived by the celebrated F. Beccaria, and mentioned by Mr. Cavallo (vol. 1. p. 241). In the middle of the receiver of an air-pump was erected a short rod, having on its extremity a metallic ball; and a similar rod proceeded through the neck of the receiver to a distance from the former exceeding four inches. If, when the receiver is exhausted, the upper ball be electrified positively, by touching the exterior extremity of its rod with the conductor of an electrifying machine, a light appears about it, but none about the lower ball. If the former be electrified negatively, by a communication with the rubber of the machine, a light will appear about the lower ball, but none about the upper. If the positive and negative Electricity were two distinct fluids, attractive of each other, light should in this experiment appear at the same time about each ball: but, on the contrary, it only appears on that ball, which according to the theory of a single fluid, possesses the greater quantity of that fluid.

conductor.

conductor. This proves that points have also the property of silently discharging the electric fluid.

These properties of pointed conductors are reconciled in this manner with the theory which has been proposed. The electric fluid, which is to be communicated to the conductor, is expanded with a greater intensity, when the surface through which it must pass is contracted. It will therefore, when a pointed conductor is presented to it, more effectually overcome any resistance which may oppose its passage; and consequently move silently and in a continued stream. For the same reason it will be in the like manner discharged from a pointed conductor.

In order to explain the application of these properties to the protection of buildings from lightning, the grand advantage hitherto derived from the knowledge of Electricity, it is necessary that the manner in which the discovery of the identity of lightning and the electric fluid was made, should be previously described.

* The correspondence of the phenomena of lightning and the electric fluid had been observed by many philosophers; but Dr. Franklin first proposed a method of verifying the hypothesis of their identity, by erecting pointed rods which should receive the electric fluid from the atmosphere, and discharge it into the earth. The discovery of this property of pointed conductors is also ascribed to Dr. Franklin. Not supposing however that

* Dr. Priestley's History of Electricity part 1. per. 9. sect. 1.

a rod of a moderate height would be sufficient for the experiment, he raised into the air, during the approach of a thunder storm, a common kite with an hempen string, by which the fluid was conducted to a key, from which it was collected and subjected to all the usual electrical experiments. The hempen string when wet conducted the fluid very copiously. Dr. Franklin furnished the kite with a pointed wire, for the purpose of drawing the lightning more effectually from the clouds; but Mr. Cavallo thinks that this does not affect the Electricity of the string, which is the most material part of the apparatus.

This discovery Dr. Franklin applied to the protection of buildings, by fixing pointed metallic rods higher than any parts of the buildings, and communicating with the earth, or rather with the nearest water. Lord Mahon, now Earl Stanhope, in his Principles of Electricity, has pointed out an effect, which he calls the *returning stroke*, to which attention must be given in the construction of these conductors.

This is the effect of that quantity of electric fluid, which, by the vicinity of a cloud highly electrified, is driven away from certain bodies, and which suddenly returns to those bodies when the cloud happens to discharge its Electricity by a stroke of lightning to any other body. Lord Stanhope has shewn experimentally, that such a returning stroke may occasion great damage, even at a considerable distance from the place where the stroke of lightning happens. The following instructions have been given

given by him for the proper construction of conductors: 1st that the rods be made of such substances, as are, in their nature, the best conductors of Electricity; 2^{ly} that they be uninterrupted; 3^{ly} that they be of a sufficient thickness; 4^{ly} that, when several rods from different parts of the building are combined together, they be perfectly connected with the common stock; 5^{ly} that they be acutely pointed; 6^{ly} that they be finely tapered; 7^{ly} that they be prominent; 8^{ly} that each rod be carried in the shortest convenient direction to the common stock; 9^{ly} that all large and prominent bodies of metal upon the top of a building should be connected with the conductor by some proper metallic communication; 10^{ly} that there be a sufficient number of high and pointed rods; and, lastly, that every part of the rods be very substantially erected.

The Electricity produced by the evaporation of water, affords an easy explanation of the origin of the Electricity in the clouds. The evaporation generally leaves the vessels negatively electrified, and therefore the vapour must carry away a portion of positive Electricity.* Clouds and rain will therefore naturally have the positive Electricity; though a cloud, when once formed, by its influence on neighbouring clouds, may cause them to become negative.

Various other natural phenomena, besides explosions of lightning, are supposed to be produced by natural Electricity. If the equilibrium of the electric matter be

* Nicholson's Introd. to Nat. Phil. vol. 2. p. 360, &c.

by any cause disturbed in the upper, and highly rarified regions of the atmosphere, it may easily be conceived, that it will be restored by coruscations similar to those exhibited in the vacuum of the air-pump. This would account for the appearance of the aurora borealis. In clear and calm weather, when the Electricity is not very strong, it may pass through the air without the assistance of any considerable quantity of vapours; and, according to the conductors which it meets in the air, it will sometimes be rendered visible during small parts of its passage, and occasion those appearances which are called shooting-stars. It has been conjectured, that the meteors called ignis fatuus consist of inflammable air, which has been kindled by Electricity. Various phenomena of water-spouts seem to be connected with the same cause. They often vanish, and presently appear again in the same place: whitish or yellowish flames have sometimes been seen moving with prodigious swiftness about them; and whirlwinds are observed to electrify the apparatus very strongly. They generally appear in those months, which are peculiarly subject to thunder-storms; and they are commonly preceded, accompanied, or followed by lightning. It is also extremely probable, that earth-quakes are caused by a discharge between a cloud and the earth. They happen most frequently in dry and hot countries, which are most subject to lightning and other electrical phenomena; and are even foretold, from the electric coruscations and other appearances in the air. They are attended by no fire, vapour, or smell, which however could scarcely fail to appear if they were occasioned by a subterraneous fire. They are usually accompanied

panied by rain, and sometimes by the most dreadful thunder-storms. But the almost instantaneous motion of the shock is the circumstance, which more especially renders it probable, that it is an effect of Electricity. The effect of an explosion of subterraneous fire would be a gradual lifting of the earth. The extensive influence of the electric fluid indicates, says Mr. Cavallo, "that Electricity must be employed for some great operation; but human industry has not yet removed the veil from this great mystery".

The effects produced by the electric fluid, when excited by electrical machines, and more especially the shocks which it gave to the human body, naturally induced an opinion, that it was possessed of the greatest efficacy in removing disorders of almost every kind. Many however of the effects, which have been ascribed to it, as an increased perspiration, or pulsation, should have been attributed to the apprehensions of the patients. This extreme admiration of the medical virtue of Electricity was naturally succeeded by an opposite persuasion; and the practical application of it was totally disregarded. At present however it is considered, says Mr. Cavallo, as "an harmless remedy, which sometimes instantaneously removes divers complaints, generally relieves, and often perfectly cures various disorders, some of which could not be removed by the utmost efforts of physicians and surgeons."

Some late discoveries concerning muscular motion have been supposed to authorize us to regard this power, as the

immediate instrument by which it is performed. Dr. Galvani of Bologna first announced these discoveries to the world in the year 1791. He discovered first, that a frog dead and skinned is capable of having its muscles brought into action by means of artificial or atmospheric Electricity: and secondly, that, independently of any artificial or natural Electricity, the same motions may be produced in the dead animal, or even in a detached limb, by merely making a communication between the nerves and the muscles, with substances that are conductors of Electricity. This latter property, which is the principal discovery, being manifested by the same substances which conduct Electricity, has been denominated Animal Electricity. In the prosecution of these discoveries a number of the most cruel experiments on animals have been performed, which no mere wantonness of curiosity could justify. On account of their importance in the advancement of useful knowledge it was necessary that they should have been made; but, their truth having been fully established, it is desirable that they should never be repeated. This property is not peculiar to a few animals only, but seems to be a property of all animals in general; a law of nature, which admits of few exceptions, and those of a very doubtful nature. The experiments have already been tried with a great variety of terrestrial, aerial, and aquatic animals; and the recently amputated limbs of an human body have been convulsed by the application of metals. The animals, which form an exception from this law, are several worms, some other insects, the oyster, and a few small marine animals. But, since the organization of these animals seems not to be possessed of
much

much sensibility, nor admits of much motion, it may be presumed that the effects of the metallic application are only too weak to be perceived by our senses. Even the living human body can be rendered sensible of the action of metallic applications, and both the senses of taste and sight may be excited by it. If a piece of metal be laid upon the tongue, and a piece of some other metal be laid under the tongue, the two pieces being brought into contact a cool and subacid taste will be occasioned, not differing much from that produced by artificial Electricity. The metals which are best adapted to this experiment, are silver and zinc, or gold and zinc. To affect the sense of sight, let a piece of zinc be placed between the upper lip and the gums, as high up as possible, and a piece of silver upon the tongue; or let a piece of silver be put high up in one of the nostrils, and a piece of gold or zinc in contact with the upper part of the tongue; and, if the metals be brought together, a faint flash of white light will appear before the eyes. But, notwithstanding the several circumstances in which these phenomena resemble those of artificial Electricity, Mr. Cavallo still doubts whether we are yet warranted in considering them as essentially of the same kind. He doubts whether the indications of attraction and repulsion, which have been said to be discovered, should not rather be attributed to other causes; and also, whether the very small quantity of the electric fluid, which could be supplied by the contact of metals of different kinds, is adequate to the production of the muscular movements. The want of the electric light, he acknowledges, may per-

haps be attributed to the very rarified state and small quantity of the fluid.

Before these discoveries had been made, the name Animal Electricity had been given to the analogous power of three fishes; namely, the torpedo, the gymnotus electricus, and the silurus electricus. Whenever a communication is made by means of substances, that are conductors of Electricity, between one part and another of any of those fishes, and a man or other animal is placed in the circuit which forms the communication, a shock similar to that produced by Electricity is perceived. The gymnotus alone affords a luminous spark, which is seen when a very small interruption is made in the circuit of communication. This power, which probably has been given to these animals as the means of defence and subsistence, vanishes with the death of the animal*.

The instruments most commonly used for exhibiting the effects of Electricity, besides the Electrical Machine already described, are the Electric Jar, the Electrometer, and the Condenser.

If a plate of glass be coated with some conducting substance, as tin-foil, on both sides, but so that the coatings shall not come very near the edge of the glass, and one of these coatings be put into either state of Electricity by the conductor of an Electrical Machine, the other

* For a conjecture concerning the means by which these fishes communicate the electric shock, see Nicholson's Journal of Nat. Phil. &c. for Nov. 1797.

coating

coating, whilst communicating with the earth, or with a sufficient quantity of conducting bodies, will be put into the opposite state of Electricity. The glass is then said to be charged. This will equally happen, if the glass be not in the form of a plate, but in any other shape whatsoever, provided that it be sufficiently thin; it being not the form, but the thickness of the glass, that makes it more or less fit to be charged. The thinner the glass is, the greater is the charge which it is capable of receiving; unless it be so thin, that the fluid shall force a passage through the glass. The glass is accordingly used in the form of a jar or phial having a rod communicating with the inside coating. The discharge is made by forming a communication with conductors between this rod and the outside coating; and, if the human body be a part of the circuit, a disagreeable sensation is experienced. The coating is used for the purpose of removing the difficulty of charging an electric plate. By its conducting nature the electric fluid is readily communicated to, or withdrawn from all the surface which it covers. The coatings must not come very near the edge of the glass, lest the Electricity should pass from the one side to the other; in which case the glass could not be charged. An instrument thus constructed is sometimes called the Leyden phial, because this property of glass was first observed at Leyden.

The cause of the charge seems to be the repulsive power of the fluid operating through the glass. In consequence of this force, when an accumulation of the fluid takes place on the one side, an equal quantity is repelled from

from the other; and when a portion is withdrawn from the one, the diminution of the resistance will permit a corresponding accumulation on the other, and will require to be supported by its increased repulsive power. These effects however cannot be produced, if the jar be insulated; and therefore an insulated jar cannot be charged. For similar reasons neither can it be discharged. The interposition of the glass prevents any direct communication between the Electricities of its sides.

* Attempts have been made to determine, by the discharge of an electric jar, the velocity of the electric fluid; but its passage has not been observed to occupy any perceivable time, though the circuit of communication was four miles in length.

A number of coated jars connected together in such a manner, that their whole force may be united, and act like that of one jar, constitute an Electrical Battery.

An Electrometer is an instrument contrived for measuring the quantity, and ascertaining the quality of Electricity. There are various kinds of electrometers, but the general principle of their construction is, that the divergence of light bodies, as of two balls of cork sus-

* Dr. Priestley's History of Electricity, part 1. per. 8. sect. 2. It may however be doubted, as Dr. Priestley has remarked, whether the same portion of the electric fluid traversed this space, and whether the shock was not rather communicated by a successive propulsion of the fluid, resembling that by which sound is propagated through the air.

pended by a string, or of two slips of gold-leaf, affords a measure of the Electricity by which it is caused. The slips of gold-leaf, which form the most sensible electrometer, are inclosed in a glass vessel, that they may not be affected by any agitation of the air; and the Electricity is communicated through the cover of the vessel. The same kind of Electricity being communicated to both parts of the electrometer, a divergence must ensue agreeably to the second law of Electricity. For ascertaining the quality of Electricity, an electrometer is electrified either positively or negatively, by communication with the cylinder or rubber of an electrical machine, and then presented to another, which has acquired the Electricity of the electrified body; for the Electricity of the body must be the same with that of the former, if the electrometers repel each other, but if they attract each other the contrary.

One defect is common to all electrometers, namely, the want of a standard of original adjustment, by means of which all such instruments might indicate the same intensity in similar circumstances.

The Condenser of Electricity has been contrived for rendering sensible a portion of Electricity, which had been too much diffused to produce any perceivable effect. It consists of a plate of brass furnished with a glass handle. The weak, and otherwise unobservable, Electricity is communicated to this plate, whilst standing on an imperfectly conducting plane, as dry marble or very dry and slightly varnished wood; and, when the plate is raised

ed from this surface, the Electricity communicated to it will be found possessed of a degree of intensity sufficient to render it sensible by its effects. The operation of this instrument is explained by observing, that the capacity of a conductor for containing the electric fluid is increased, when it is presented near another conductor not insulated. The increase of capacity will be evident, if an electrometer be annexed to the former; for the divergence will be diminished when it is brought near the latter, and restored when it is removed, excepting so much as may correspond to the Electricity imparted to the air during the experiment. The plate of brass will therefore receive a greater quantity of the fluid, whilst it stands upon the imperfectly conducting surface; and will consequently exhibit it in a state of greater intensity, when raised from that surface.

The increase of capacity seems to be the effect of a condensation of the fluid belonging to the plate, occasioned by the vicinity of the imperfectly conducting surface. If the Electricity of the plate be positive, it will be accumulated at the contiguous surface of the plate, and an opportunity will be afforded for the introduction of another portion; and if it be negative, it will be repelled to the opposite surface, and from it thus accumulated another portion may be withdrawn. The surface, to which the plate is applied, must not be a good conductor, lest, on account of its contiguity, it should destroy the Electricity of the plate; nor must it be a non-conductor, because in this case its electric fluid could not undergo those changes, which are necessary to the operation.

* Various

* Various instruments of a complicated construction have been contrived for rendering sensible quantities of Electricity still smaller than those which may be discovered by the condenser. By these instruments it is made evident, that there is scarcely any friction, any evaporation, or condensation, which does not excite Electricity; and consequently, that there is a continual motion of the electric fluid throughout the works of nature.

* Nicholson's Journal of Nat. Phil. &c. for Dec. 1797.

The magnetic properties of the various substances are classified into two groups, diamagnetic and paramagnetic. Diamagnetic substances are those which are repelled by a magnetic field, and paramagnetic substances are those which are attracted by a magnetic field. The magnetic properties of a substance are determined by the arrangement of its atoms and molecules. In diamagnetic substances, the electrons are paired, and the magnetic moments of the individual atoms cancel each other out. In paramagnetic substances, the electrons are unpaired, and the magnetic moments of the individual atoms do not cancel each other out. The magnetic properties of a substance are also determined by the temperature. In general, the magnetic properties of a substance are more pronounced at lower temperatures.

CHAPTER

162
CHAP. IV.

OF MAGNETISM.

THE word Magnetism has been supposed to be derived from the name of a shepherd, who is said to have first discovered the magnet, or load-stone, on mount Ida. The natural magnet is an ore of iron, and contains a greater quantity of iron than most other iron ores. An artificial magnet is a portion of some ferruginous substance, to which the magnetic properties have been communicated. Every magnet, whether natural or artificial, is possessed of the five following properties: 1st the power of attracting ferruginous substances; 2^{ly} polarity; 3^{ly} the power of attracting and repelling any other magnet; 4^{ly} inclination; and 5^{ly} the power of communicating the magnetic properties.

The substance of this chapter has been taken from Cavallo's Treatise on Magnetism.

1st. A magnet

1st. A magnet attracts a piece of soft iron more forcibly than any other ferruginous body of the like shape and weight. The attraction is strongest near the surface of the magnet, and diminishes as the distance is increased; but the law of the diminution has never been ascertained. Heat weakens, or destroys, the power of the magnet. From this cause therefore, besides others which may concur, the power of a magnet must be continually varying. It has been observed, that amongst the natural magnets the smaller generally possess a greater attractive power than in the proportion of their magnitudes. Neither the attraction nor the repulsion of Magnetism is sensibly affected by the interposition of any bodies except those of a ferruginous nature. Thus if, when a magnet is placed at the distance of an inch from a piece of iron, the attraction be able to counterbalance the weight of an ounce, the attraction will still be found to counterbalance exactly the same weight, though a plate of other metal, or glass, or any body not ferruginous, be interposed between them; or though they be inclosed in separate boxes. Neither the absence nor presence of air has any effect upon it. It must however be observed, that Mr. Cavallo has, by some new experiments, discovered that brass, by hammering, is rendered magnetic, independently of any particles of iron which it might be supposed to contain. To prove that this was not caused by particles of iron, he inclosed a portion of crocus martis, which is a calx of iron, in a piece of brass to which hammering had not communicated Magnetism; and he found that neither did it acquire this power from hammering after that the calx had been inserted. Hence

he concluded, that the magnetic power communicated by hammering to other pieces, was not derived from any efficacy of the hammering in reducing any particles of iron to a metallic state. This conclusion he confirmed by observing, that the same piece of brass, having been made red-hot, discovered when cooled a magnetic power only in that part in which the calx of iron was contained. If therefore the Magnetism of brass were derived from any ferruginous matter contained in it, the action of fire should render it magnetic, contrary to experience. Platina he also observed to acquire Magnetism, like brass, from hammering, and to be in the same manner deprived of it by heat.* It is indeed true, that most bodies, either naturally or by being subjected to the action of fire, may be rendered in some measure capable of being attracted by the magnet "so general," says Mr. Cavallo, "is the dispersion of iron, or such is the tendency which most bodies have towards the magnet."

2ly. "When a magnet is placed so as to be at liberty to move itself easily, it turns one, and constantly the same, part of its surface towards the north pole of the earth, or towards a point not much distant from it; and consequently the opposite part of its surface towards the south pole of the earth, or towards a point not much distant from it. These parts on the surface of a magnet are therefore called its Poles, the former being denominated its North Pole, and the latter its South Pole. The

* M. de Humboldt discovered a very singular instance of magnetic polarity in a mountain of Serpentine in Germany. Nicholson's Journal of Nat. Phil. &c. for June 1797.

property itself is called the magnets Directive Power, or Magnetic Polarity; and when a magnetic body places itself in that direction, it is said to Traverse. A plane perpendicular to the horizon, and passing through the poles of a magnet when standing in their natural direction, is called the Magnetic Meridian: and the angle made by the magnetic meridian and the plane of the meridian of the place where the magnet stands, is called the Declination of the magnet."

The polarity of the magnet, which was not known in Europe before the thirteenth century, is the property which renders it most interesting to mankind. By it they are enabled to guide themselves where no distinguishable marks would otherwise acquaint them with their true course. Mariners out of sight of land, travellers in a desert, and miners beneath the surface of the earth, can all determine, by the assistance of this property, how they may proceed in any proposed direction. This however must be received with some limitation. The poles of the magnet are seldom observed to point exactly to those of the earth, and generally decline a few degrees to the eastward or westward. This declination, which is also called the Variation, is different in different places, and is continually changing in the same place. It has not yet been discovered that it is subject to any law, or comprized within any determined period. When the variation was discovered by Columbus in his first voyage to America in year 1492, it was easterly at London; in the year 1657 there was at London no variation; and it is now about 22 degrees westerly. It has been proposed, to observe the variation

riation for the purpose of ascertaining the longitude; but these observations seem to be destitute of the regularity necessary for this investigation. Captain Vancouver, in the account of his late voyage round the world, has given an instance of the great uncertainty of this matter. * He has remarked, that, if he had depended upon this method of finding the longitude, he might have searched in vain for the Cape of Good Hope, since an error was actually discovered of nearly 6 degrees of variation. "The observations for the variation were made," he says, "with the greatest care and attention; and though generally considered as very correct, they differed from one to three, and sometimes four degrees, not only when made by different compasses placed in different situations on board, and the ship on different tacks, but by the same compass in the same situation, made at moderate intervals of time, the difference in the results of such observations at the same time not preserving the least degree of uniformity."

There is also, besides the variation already mentioned, a daily variation, which is evidently influenced by heat and cold. † Mr. Canton observed the diurnal variation at London between the years 1756 and 1760 during 603 days, and on 574 of them found it regular; that is, he found that the westerly variation was increasing from about eight or nine o'clock in the morning till about one or two in the afternoon, when it became stationary for some time; and that it then decreased, and returned to its for-

* Vol. I. p. 15.

† Phil. Trans. vol. 51. part 1.

mer situation, or near to it, in the night, or by the next morning. The irregular variations were always accompanied by an aurora borealis. He accounted for this variation in the following manner. The attractive power of a magnet, whether natural or artificial, decreases whilst the magnet is heating, and increases whilst it is cooling. The magnetic parts of the earth in the north, on the eastern and western sides of the magnetic meridian, equally attract the north end of the magnetic needle. When therefore the eastern magnetic parts are heated faster by the sun in the morning, the attraction of the western parts must prevail. When the attracting parts of the earth on each side of the magnetic meridian have their heat increased equally, the needle will be stationary. But when the western magnetic parts are either heating faster, or cooling more slowly than the eastern, the needle will move eastward. And when the eastern and western parts are equally cooled, the needle will again be stationary. The irregularity of the diurnal variation Mr. Canton ascribed to subterranean heat, generated without any periodical regularity. The aurora borealis, which was observed at the same time, he supposed to be the Electricity of the heated air above the earth; and this, he remarked, would appear chiefly in the northern regions, as the alteration in the heat of the air would there be greatest. Conformably to this solution, he found that the mean diurnal variation for each month was greatest in June, and least in December. In the former month it was $13^{\circ}, 21''$; in the latter $6^{\circ}, 58''$.

31y. If the north pole of one magnet be presented to the south pole of another, they will be attracted towards each

each other; but if two similar poles be presented to each other, a repulsion will take place. There are some apparent exceptions from this law. It often happens, that magnets attract each other when poles of the same kind are presented to each other; and sometimes neither attraction nor repulsion is observed. They may however be reconciled with the general property. It will be shewn, in treating of the 5th property of magnets, that when a piece of any ferruginous substance is brought within a certain distance of a magnet, it become itself a magnet, and that part of it, which is nearest to the magnet acquires a contrary polarity. If therefore one of the two magnets be more powerful than the other, it may change the pole of that magnet, in the same manner in which it gives Magnetism to any other ferruginous substances exposed to its influence, and then an attraction will take place apparently between poles of the same kind, though really between poles of different kinds. If the one magnet be gradually brought near to the other, there must be a certain point at which the repulsion ceases and the attraction begins, and consequently at which neither can take place. A magnet is often observed to have more than two poles. Their number, nature, and situation are ascertained by presenting the various parts of the surface to one of the poles of another magnet freely suspended; for those parts of the magnet, which repel that pole, have the same polarity, and those which attract it, have a different polarity. The attractive and repulsive power is not confined to the poles of a magnet, but it is strongest in those points, and in the middle points between

between them there is neither attraction nor repulsion.

24y. If a small oblong magnet be suspended freely in such a manner, that it may be in an horizontal position when not disturbed by another magnet, and be brought just over the middle of a larger magnet whose poles are in a line parallel to the horizon; it will direct its south pole towards the north pole of the other, and its north pole towards the south pole of the other, but will continue in an horizontal position. If the small magnet be moved a little towards either pole of the larger magnet, its nearer pole will be inclined downwards; and this inclination will become greater, according as the smaller magnet approaches to the pole of the larger. If the small magnet be brought to that pole, it will place itself in the same right line with the axis, or right line joining the poles, of the larger magnet. For applying these observations to the general property of magnetic bodies, it is only necessary to conceive that the earth itself is the larger magnet. Thence it will follow, that a magnetic body should have no inclination in regard to the horizon in those parts of the earth which are equidistant from the magnetic poles of the earth; that its inclination should increase as it approaches either of these poles; and that at either pole it should place itself in a vertical position. That the earth may be considered as a large magnet, is proved from the two following considerations: 1st. almost all the magnetic phænomena may be exhibited with the earth, as far as it may be tried; and 2^{ly}. vast masses of

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ferruginous

ferruginous matter actually magnetic are dug out of almost every part of the earth.

Mr. Cavallo has remarked, that the only magnetic phenomenon, which has not been observed to belong to the earth, is the attraction of ferruginous substances. If a piece of iron be presented to either of the poles of a common magnet, it will be powerfully attracted; but if it be presented to the parts equidistant from the poles, the attraction will be considerably weaker. It might accordingly be expected, that the weights of the same piece of iron at the equator and near either pole should be sensibly different, since the increased attraction of Magnetism would near the pole cooperate more powerfully with the general attraction of Gravitation. Such a difference has not been observed. Mr. Cavallo however contends, that we should not therefore infer, that the earth does not exert any magnetic attraction towards the iron on its surface, and that this attraction is not stronger near the poles than near the equator; because, the Magnetism of the earth being very weak, the difference of attraction in different places must be likewise very small, though the directive power, which extends to a greater distance than the former, is considerably strong. It must however be remarked, that the weight of the iron, independently of magnetic attraction, should be greater near the pole, agreeably to the third law of Gravity mentioned in page 76.

The inclination or dipping of the magnet has not been found more capable of being reduced to rule than its variation.

variation. It is continually changing in the same place, and its changes in different places are very unequal; so that it can only be ascertained by actual experiment. The inclination in London in the year 1775 was 72° , 3'.

5ly. When any ferruginous body is presented to a magnet, within a proper distance from one of its poles, it becomes instantly magnetic, the part of it, which is nearest to the magnet, acquiring the contrary polarity. This acquired Magnetism is stronger according as the body approaches to the state of soft iron; but its permanency observes the opposite rule. The hardest steel retains it for many years with little or no diminution; whereas very soft iron loses it intirely the moment in which it is removed from the influence of the magnet.

Magnetism may apparently be communicated without the aid of a natural magnet; but in this case the magnetic power is communicated from the earth, which must be considered as a magnet. It is therefore universally true, that no Magnetism is communicated except by the action of a magnet.

To communicate Magnetism directly from the earth, let a straight bar of soft iron be placed in a vertical position, or rather in the angle of magnetic inclination, and it will become magnetic, the lower extremity being a north pole, and the upper a south pole. In the southern parts of the earth the lower extremity would become a south pole. A bar four or five feet long and more than an inch thick, will in this position be capable of attracting a small

bit of iron, or a common sewing needle. If the bar be placed in a direction perpendicular to that of the magnetic inclination, it will not acquire Magnetism; because in such a situation the actions of both magnetic poles of the earth upon the extremities of the bar are equal. If the bar be placed in any intermediate position, it will acquire a greater or less degree of magnetic power; according as it approaches to either of the preceding situations. A bar of hard steel, or of hard iron, will not acquire any Magnetism from the earth, because the magnetic power of the earth is not sufficiently strong.

The following laws have been observed to prevail in the communication of the magnetic power. 1st. A magnet, by communicating Magnetism to other substances, not only loses nothing of its power, but is rather improved. 2^{ly}. A magnet can never communicate a greater power than itself possesses, or even an equal degree of it; but several magnets, of nearly an equal degree of Magnetism, can communicate conjointly a stronger power than one of them singly. 3^{ly}. The communicated Magnetism is strongest, when the ferruginous body is applied in contact with the magnet. 4^{ly}. For communicating to any body the greatest degree of Magnetism, of which it is capable, it must be exposed to the influence of the magnet for a considerable time; and this time is greater according as the body is harder. 5^{ly}. The communicated Magnetism, other circumstances being similar, is more permanent according as the body is harder; and also, within certain limits, * according as the body is more oblong. The

* The most advantageous shape has been observed to be that, in which

The communication of Magnetism is facilitated by heating, drilling, filing, hammering, and by the electric stroke. It is facilitated by heating, because the iron is thereby rendered softer. The other processes seem to facilitate it by exciting heat and a vibratory motion of the parts of metal. If a powerful magnet be struck by the electric shock, its power will be diminished; and it may in general be observed, that the same means, which facilitate the communication of Magnetism, when pieces of iron are properly situated in regard to the poles of the earth, do also facilitate its loss, when magnets are improperly situated.

The most convenient method of communicating Magnetism is the following, in which two bars already rendered magnetic are employed. Let the bar, to which Magnetism is to be communicated, be *AB* (plate 4. fig. 2.) and let the marked extremity *A* be that which is required to become the north pole. Place the two magnetic bars *CD*, *EF* perpendicularly upon *AB*, and at small and equal distances from its middle point; so that the bar *CD*, which is nearer to *A*, may have its south pole in contact with *AB*, and the other *EF* may have its marked extremity, or north pole, in contact with it. Then let the two magnetic bars be slid alternately towards *A* and *B*, being stopped when *CD* arrives at *A*, or *EF* at *B*; and when the magnetic bars are to be removed, let them be previously brought to the middle point of *AB*. When the magnetic bars are powerful, and the

which the length is about ten times greater than the breadth, and twenty times greater than the thickness.

bar

bar *AB* is of very good steel, and not very large, a dozen of strokes are fully sufficient,

When the bar *AB* is large, and it is required to give it the greatest possible power, the effect of the magnetic bars may be improved by the following expedients. 1st. Let the magnetic bars be joined at their upper extremities, a piece of some substance not ferruginous being interposed between them, as in plate 4. fig. 3. By this contrivance the upper poles of these bars will strengthen each other and consequently their lower poles will also be strengthened. 2^{ly}. Let the bar *AB* be placed between two bars of soft iron, or two other magnets, as in the figure. If two magnets be used for this purpose, the poles adjacent to the bar *AB* should be the contrary poles to those, which it is designed to communicate to that bar.*

The power of a magnet, whether natural or artificial may be considerably increased by a gradual augmentation of a quantity of iron suspended by it. This is explained by observing, that the iron, being rendered magnetic, strengthens the Magnetism of the magnet. When the iron falls from the magnet, the latter will lose a part of its acquired power, which must be restored gradually as before. A magnet for the same reason, if no iron be affixed to it, will lose much of its power.

* A farther improvement of the method of making artificial magnets is described in Mr. Nicholson's Journal of Nat. Phil. &c. for May 1798.

A natural

A natural magnet is commonly inclosed in an apparatus, which is called its Armature. This consists of two plates of soft iron, applied to those surfaces in which the poles are situated, and projecting a little on one side; as in plate 4. fig. 4. Those projecting parts are made much narrower than the plates. The armature is secured by a box of some other metal, as brass. The advantage of it is, that the poles are thereby brought to act in the same plane, so that a bar of iron may be affected by both at the same time. It however rather strengthens the power of the magnet, for the same reason for which any piece of iron, applied to a magnet, tends to render it more powerful. What has been said of natural, is also applicable to artificial magnets. Many magnetic bars may be joined together and armed, so as to form a very powerful compound magnet*.

The invention of the Mariner's Compass is the great benefit derived from our knowledge of Magnetism. A slender magnetic bar, or needle, is supported so that it may move freely, and balanced so that it may, notwithstanding the magnetic inclination, continue horizontal. To the needle is attached a card, on which is a circle concentric to the point of suspension, whose circumference is equally divided in 32 points, called the Points of the Compass, or Rhumbs. The box containing the card and needle is made of a circular form, and either of wood or brass. It is suspended within a square wooden box, by means of two concentric rings, called Gim-

* For a description of such a magnet, see Nicholson's Journal, *ibid.*

bals, so fixed by transverse points of suspension, that the inner one, or compass-box, shall * as much as possible retain an horizontal position in all motions of the ship. The compass-box is covered with a pane of glass, that the motion of the card may not be disturbed by the wind. The suspension of the needle is best performed by inserting a piece of agate into it, and making in the stone a conical cavity, which may rest on the sharp point of a perpendicular wire.

For ascertaining small degrees of magnetic attraction a still more free suspension is necessary. Mr. Cavallo for this purpose suspended the needle by a chain consisting of five or six links of horse-hair. He some times placed the needle floating on quicksilver; but this method, though very sensible, is inconvenient. The quicksilver, in a short time after that it has been poured out, contracts a crust which impedes the motion of the needle, and which can only be removed by pouring the quicksilver through a slender funnel of paper.

In several respects there is a remarkable correspondence between the powers of Electricity and Magnetism, whilst in others there is as remarkable a disagreement. 1st There are two opposite states of Electricity; and bodies in similar states repel each other, whereas an attraction takes place between those which are in opposite states. In the like manner there are in Magnetism two

* For some observations on this subject, see Nicholson's Journal for December 1797.

contrary polarities ; and those parts of bodies which are possessed of the same polarity repel each other, but an attraction takes place between those of different polarities. 2^{ly}. One sort of Electricity is accompanied by the other. Thus, if a plate of glass be electrified positively on the one surface, it must have the contrary Electricity on the other. In the same manner the one magnetic polarity accompanies the other ; for no body is observed to be possessed of one alone. 3^{ly}. The electric power may be retained by certain bodies, called electrics ; but easily pervades other substances called conductors. The magnetic power also is retained by ferruginous substances, especially those of an hard nature : but it pervades, without the least perceivable impediment, all other substances. On the contrary the following differences are observable between Electricity and Magnetism. 1st. The magnetic power does not like the electric, affect our senses with any light, smell, taste, or noise. 2^{ly}. The electric power attracts bodies of every sort, and not merely those of a ferruginous nature. 3^{ly}. The electric power appears to reside at the surface of electrified bodies, but the magnetic to be internal. 4^{ly}. A magnet does not, like an electrified body, lose any part of its own power by communicating it to other bodies. 5^{ly}. The propagation of the electric power is instantaneous ; but that of the magnetic power requires some sensible time. Thus, if a short piece of iron of about 40 or 50 lbs. weight be placed near one of the poles of a magnetic needle, so as to draw it a little out of its natural direction ; and one pole of a strong magnet be applied to the farther end of the

A a iron ;

iron; some seconds will elapse before the needle will be affected by it.

Various hypotheses have been devised for explaining the phenomena of Magnetism, but they are all attended by considerable difficulties. The analogy subsisting between Electricity and Magnetism induced *Æpinus* to imagine a magnetic fluid similar to that supposed in the theory of positive and negative Electricity. Agreeably to this hypothesis iron, and all ferruginous substances, contain a quantity of magnetic fluid, which is equally dispersed through their substance, when those bodies are not magnetic: but when the magnetic fluid is driven to one end, the body becomes magnetic, one extremity of it being now overcharged with magnetic fluid, and the other undercharged. The attractions and repulsions of Magnetism may accordingly be explained in the same manner as those of Electricity.

Our knowledge of Magnetism has not yet afforded a theory which could enable us to ascertain the variation of the needle for any proposed place or time. To explain in general the cause of the variation, and of its continual changes, *Dr. Halley* supposed that the earth consists of two distinct parts, an external shell, and an internal nucleus or globe separated from the surrounding shell, but having the same center of Gravity. Each of these parts he regarded as a magnet endued with two poles; and conceived that the diurnal motion of the earth had been impressed from without, and that the velocity of the nucleus must consequently seem to revolve slowly to the westward,

westward, and its poles to revolve in lesser circles round the poles of the intire body of the earth; and that as the relative position of the four magnetical poles and the poles of the earth is changed, so must the direction of the needle be altered. Mr. Cavallo however appears to have given a more satisfactory solution. He observes that the attraction subsisting between magnetic bodies is subject to variations occasioned by several causes: that it is increased by cooling, by the reduction of the calx of iron to its metallic state, and by the action of acids upon iron; and that it is diminished by heat, and the decomposition of iron. He remarks also that the earth contains almost every where ferruginous bodies, and that considerable alterations in the state of these bodies must continually take place, by irregular heating or cooling, by volcanos, earthquakes, and the chemical action of the internal parts of the earth. The magnetic needle must necessarily be affected by these changes, because its direction must be considered as determined by the combined action of all the magnetic bodies contained in the earth.

Many have believed, that there is a circulation of a magnetic fluid through the poles of a magnet. This opinion has been occasioned by observing the regular arrangement of filings of iron attracted by a magnet. Thus, if some filings of iron be strewed upon a sheet of paper laid on a table, and a small artificial magnet be placed among them; the table being a little agitated, so as to move the filings, they will dispose themselves round the magnetic bar in regular arches reaching from the one

pole of the magnet to the other. This phenomenon was supposed to indicate the course of the magnetic fluid; but the true cause is that each particle of the filings becomes actually a magnet, and according to the 4th property of magnetic bodies, will, when at liberty to obey the magnetic power, dispose itself in a direction corresponding to its situation in regard to the poles of the larger magnet, or to the adjacent particles. Those particles therefore, which are contiguous to the poles of the larger magnet, will stand vertically; these being rendered magnetic, will attract other particles; and so series of small magnets will be formed, decreasing in power as they recede from the larger magnet. As each of those particles has two magnetic poles, the farther extremities of those, which form the series proceeding from the north pole of the large magnet, will have the north polarity; and the farther extremities of those proceeding from the south pole will have the south polarity. Hence, when these series meet, they attract each other, and the arches are consequently completed.

The magnetic power has been supposed to possess considerable virtue in removing the diseases of the human body, but the imposture was detected by a committee consisting partly of Physicians and partly of Academicians, over whom Dr. Franklin presided. Their conclusion was, that it was not intirely useless even to philosophy; as it is one fact more to be consigned to the history of the errors and illusions of the human mind, and a signal instance of the power of imagination.

C H A P.

C H A P. V.

OF IMPULSE.

BESIDES those active forces which generate motion, it is necessary to consider that, by which it is communicated from one body to another. This may be in general called Impulse. When both bodies had been previously in motion, it may be called Collision. For determining its effects bodies are conveniently distinguished into non-elastic and elastic; though perhaps there is not any body perfectly correspondent to either description. Bodies are said to be non-elastic, which, when compelled to change their shape, do not exert any force to return to it again. Bodies are said to be elastic, whose parts yield to a force impressed, and endeavour to restore themselves to their original position when that force has been removed; and they are said to be perfectly elastic, when

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the restitutive force is equal to the impressed force, so that the parts return to their former situation.

Experiments relating to Impulse being most conveniently made with pendulous bodies, it is necessary to premise the following lemma, by which their velocities may be compared.

Lemma. The velocities which a pendulous body acquires in descending along different circular arches to the lowest point, are proportional to the chords of the arches.

Let P (plate 4. fig. 5.) be a pendulous body suspended from the center C ; and let it descend to the lowest point P through the different arches AP , BP . The velocities acquired will be proportional to the chords AP , BP . For, the horizontal lines AH , BK being drawn, the velocities acquired in descending along the arches, will be respectively equal to those which might be acquired in falling down the heights of those arches HP , KP . (ch. 1. sect. 3. prop. 5. cor.) But the latter velocities are proportional to the chords. For, by the property of motions produced by constant forces, the velocities acquired in falling down the heights HP , KP , are in the subduplicate ratio of these heights; and it may be shewn, that the chords are in the same subduplicate ratio. The square of the chord AP is equal to the rectangle contained under HP and the diameter DP (Euch. book. 6. prop. 8. cor. and prop. 17.); and the square of the chord BP is in the like manner equal to the rectangle under the diameter and KP . Therefore the square of

AP

AP is to the square of BP , as the former rectangle to the latter; and consequently (Euch. book 6. prop. 1.) as AP to KP . Therefore the chord AP is to the chord BP in the subduplicate ratio of HP to KP , that is, in the ratio of the velocities.

This being premised, let two equal balls of soft clay, or putty, which may be considered as non-elastic, be suspended in such a manner from two points by two equal threads, that the threads may be parallel, and the centers of the balls in a right line parallel to the horizon.

Case 1. If, whilst one of the balls remains quiescent, the other be raised along an arch, whose chord is of any assigned length, as 6 inches, and then suffered to descend so as to strike the former; the two balls, being non-elastic, will not separate after the impulse, but will ascend together through an arch, * whose chord is equal to the half of the former.

In this case the velocity of the compound body is one half of that of the striking body (lemma), the velocities being proportional to the chords. Since therefore the quantity of motion is proportional to the quantity of matter and velocity conjointly (schol. 2. to the laws of

* For the general solution of these cases, whether the quantity of matter contained in the bodies be equal or unequal, see Hellsam's Lectures in Natural Philosophy, p. 58; or Wood's Principles of Mechanics, art. 198. and for the method of making allowance for the resistance of the air, see Phil. Nat. Princ. Math. schol. ad leges motus.

motion);

motion); and the quantity of moving matter is doubled after the impulse, and the velocity reduced to one half; the quantity of motion is the same as before. But this quantity is now divided between the two equal bodies. The striking body has therefore lost one half of its motion, and the other body has acquired one half.

Case 2. If the balls be at the same instant let fall through unequal arches in the same direction, as through arches whose chords are of 9 and 5 inches, they will ascend together in the same direction, and the chord of the arch described in the ascent will be the arithmetical mean between those chords, or of 7 inches.*

In this case the velocities before the stroke being as the numbers 9 and 5 (lemma), and after the stroke the common velocity being as 7, the body descending through the greater arch has lost 2 parts of its velocity, and consequently of its motion, and the other has acquired just so much.

Case 3. If the balls be at the same instant let fall in opposite directions through equal arches, they will after collision continue quiescent.

In this case the velocities being equal (lemma), and the quantities of matter also equal, the quantities of motion

* In the experiments illustrating this and the 4th case there is an inaccuracy resulting from the inequality of the times, in which the unequal arches are described, since the bodies do not meet in the lowest point.

are equal. The quiescence of the bodies proves that these equal quantities of motion destroy each other.

Case 4. If the balls be at the same instant let fall through unequal arches in opposite directions, as through arches whose chords are of 9 and 5 inches, they will ascend together in the direction of the body which descended through the greater arch, and the chord of the arch described in the ascent will be the half of the difference of the chords of the arches previously described, or 2 inches.

In this case the velocities before the collision being as the numbers 9 and 5 (lemma), and after the collision the common velocity being as 2, the body descending through the greater arch has lost 7 parts of its velocity, and consequently of its motion; and the other has undergone an equal and contrary change, having lost its original velocity which was as 5, and received a velocity proportional to 2 in the opposite direction.

The effects of the Impulse of elastic bodies must be different from those of the Impulse of such as are non-elastic, because the elastic force, in restoring the compressed parts to their former situations, must tend to separate the bodies from each other. * Mr. Atwood has illustrated the operation of the elastic force, by supposing a spring of a spiral form, whose axis coincides with the

* Analysis of a Course of Lectures in Natural Philosophy, Art. 56.

direction of motion, to be interposed between the two bodies. In whatever degree the spring is compressed, it will at the same instant exert the same elastic force on both bodies. When the spring first begins to be compressed, the force, by which it retards the striking body and accelerates the body struck, is the least. It will afterwards continually increase until the compression is the greatest possible, which will be when the two bodies have a common velocity; and will then begin to restore itself to its former state. If the restitutive force be equal to that of compression, the action of the spring will be similar to that of perfectly elastic bodies. In the Impulse of such bodies on each other it is obvious that the quantities of motion communicated and lost must be just double of those, which are communicated and lost in the corresponding cases of non-elastic bodies; because the effects of an equal force are superadded to those of the mere Impulse: and in every degree of imperfect elasticity, these additional effects must be proportioned to that degree. The results therefore of the Impulse of perfectly elastic bodies may be predicted, by determining first what would be the results if the bodies were non-elastic, and then doubling the quantities of motion communicated and lost. The experiments however must differ in some degree from this theory, because no bodies perfectly elastic are known.

Case 1. If two equal balls of ivory be suspended in the manner already described, and one of them be suffered to descend through any arch, so as to strike the other;

other; the latter will ascend through an equal arch, and the other will remain at rest.

In the 1st case of non-elastic bodies, the striking body lost the half of its motion, and the other acquired that quantity. In this case, agreeably to what has been mentioned in the preceding paragraph, the effects are doubled; since the striking body loses its whole motion, and the other acquires an equal quantity,

Case 2. If the balls descend in the same direction, through unequal arches, as through arches whose chords are of 9 and 5 inches, they will ascend in the same direction, but with interchanged velocities.

In the 2nd case of non-elastic bodies, the balls ascended together through an arch, whose chord was the arithmetical mean between the chords of the arches previously described, and the quantities of motion communicated and lost were the halves of the difference of the original motions. In this case the quantities of motion communicated and lost are each equal to the whole difference.

Case 3. If the balls be let fall through equal arches in opposite directions, they will both be reflected by the stroke, and will ascend to the same heights from which they fell.

In the 3rd case of non-elastic bodies, the two balls remained quiescent after the stroke, each having lost its

whole motion. In this case each body receives an equal quantity of motion in the opposite direction.

Case 4. If the balls descend in opposite directions through unequal arches, as through arches whose chords are of 9 and 5 inches, they will be reflected with interchanged velocities.

In the 4th. case of non-elastic bodies, the balls ascended together through an arch whose chord was of 2 inches, and each lost 7 parts of motion. In this case the body descending through the greater arch loses 7 parts more of motion, being reflected through an arch whose chord is of 5 inches; and the other body also suffers an equal additional loss, being reflected through an arch whose chord is of 9 inches.

It may be remarked, as a general observation on all these cases, that they afford a confirmation of the third law of motion; since in every instance the opposite changes produced in the states of the two bodies are equal. The quantity of motion communicated from one body to another, which is either quiescent or moving in the same direction, is equal to that lost by the striking body; and the quantities of motion lost by two bodies moving in opposite directions are also equal. Hence it follows, that the sum of the motions of two bodies moving in the same direction, or the difference of the motions of two bodies moving in contrary directions, is the same after the stroke which it had been before.

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The account which has been given of the operation of the elastic force, will explain what at first seems paradoxical; that a body may communicate to another a greater quantity of * motion, than that which itself had possessed. This must happen, if the two bodies be perfectly elastic, and the body struck contain the greater quantity of matter. For, if the bodies were non-elastic, the same quantity of motion, which had been possessed by the striking body, would be communicated to the compound formed of the two bodies; and the body struck, being greater than the other, would possess more than the half of the motion. Consequently, the effects being doubled by elasticity, it must acquire more than the whole motion.

Although the effect of Impulse depends in general on the quantity of motion, the same effect will not in all cases result from the same quantity of motion. A pistol bullet shot through a door may not communicate to it motion sufficient for overcoming the friction of the hinges; and yet a larger piece of lead with a velocity in the same proportion less, and consequently with the same quantity of motion, may shut the door, although the striking part be not larger than the bullet. * The moti-

* It must however be observed, that the communicated velocity will be less than that of the striking body. For the velocity of the compound, if the bodies were non-elastic, would be to that of the striking body, as the striking body to the compound (see note to case 1. of non-elastic bodies), and consequently less than the half of that velocity. When therefore it has been doubled by elasticity, it will still be less than the whole.

* See Atwood's Treatise on the rectilinear Motion and Rotation of Bodies, sect. 3. prop. 11.

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on communicated to the door by the bullet must be determined by that which the bullet loses; and, on account of the great velocity of the bullet, the motion lost in passing through so small a space as the thickness of the door may be insufficient to overcome the friction, by which the motion of the door is resisted.

ly elastic, and the body of matter. For if the bodies were non-elastic, the loss of motion, which had been possessed by the striking body, would be transmitted to the body struck, and the two bodies, and the body struck, being formed of the two bodies, and the body struck, being greater than the other, would possess more than the half of the motion. Consequently, the effects being doubled by division, it must acquire more than the whole motion.

Although the effect of impulse depends in general on the quantity of motion, the same effect will not in all cases result from the same quantity of motion. A pistol bullet fired through a door may not communicate to it motion sufficient for overcoming the friction of the hinges; and yet a larger piece of lead with a velocity in the same proportion less, and consequently with the same quantity of motion, may move the door although the first might be not larger than the latter. * The next

* It will however be observed, that this communicated velocity will be lost, from that of the striking body. For the velocity of the striking body, if the bodies were non-elastic, would be lost to that of the latter body, as the first body is the compound of the two, and it is of non-elastic bodies, and consequently, its loss is the loss of the velocity. I have therefore in this chapter, supposed it will be the

C H A P.

Of the various kinds of Motion and Resistance of

C H A P. VI.

OF THE

MECHANIC POWERS.

TO this account of the natural forces, by which motion is either generated or communicated, it is farther necessary to subjoin an explanation of the principles of those artificial forces, or Mechanic Powers, by which it is most conveniently applied to the several uses of human life. It must however be remembered, that these do not in reality constitute any new powers. Their efficacy consists only in accumulating a greater quantity of some natural power, and directing it to the attainment of some proposed end. This is evident from the consideration, that whatever is gained in power by any species of machine, is lost in time. The same quantity of motion might therefore be produced in the same time without the machine, if it were equally convenient to produce the effect proposed in successive parts, as in the whole

whole collectively ; but, because it frequently happens, that the intire effect must be produced together, it is necessary to have recourse to mechanical contrivances, for accumulating and combining those separate and successive exertions of power. A man could with a single rope move in a certain time the same quantity of matter, which he could move in the same time by a machine, but he could do this only in detached parts. The machine enables him to move at once the whole mass, to which his unassisted strength would be utterly inadequate. There are six simple machines, or Mechanic Powers, the lever, the pulley, the inclined plane, the wheel and axle, the wedge, and the screw ; and the more complicated engines are only various combinations of these simple contrivances. Though the design of the Mechanic Powers is to communicate motion, the case of equilibrium is that which is investigated in the consideration of their properties, because, this being ascertained, an addition of force will overcome the resistance.

A lever is an inflexible bar, moveable about a point, which is called the fulcrum or prop.

The property of the lever cannot be deduced immediately from the general principles of motion stated in the Introduction, because the forces acting upon it are not applied at the same point, which those general principles suppose. Various methods of obviating this difficulty have been proposed ; but that lately suggested by

* Mr. Vince seems to be the most satisfactory. His

method consists in demonstrating a principle assumed for this purpose by Archimedes, which he effects by the assumption of the following principle as self-evident.

Axiom. If a lever, as QP (plate 4. fig. 6.) rest upon two fulcrums Q and P ; and two equal bodies A and C , supported by the lever, be similarly situated in respect of its extremities, so that AQ may be equal to CP , and AP may be equal to CQ ; the two fulcrums will be pressed by equal weights, and each will sustain the half of the sum of the weights of the bodies A and C .

Lemma. If two equal weights act perpendicularly upon a lever, the effort to put it in motion will be the same, as if they acted together at the middle point between them.

Let AP (plate 4. fig. 7.) be the lever moveable about P , and let A and C be two equal bodies supported by it. Conceive PA to be produced to Q , so that AQ may be equal to PC , and the extremity Q to be sustained by another fulcrum. In this case, by the preceding axiom, the fulcrum at Q would sustain one half of the sum of the weights of the two bodies. If therefore these bodies were removed, and a weight equal to that sum were placed at B the middle point between them, since the point B also bisects the whole QP , the pressure upon the fulcrum at Q would be the same as before; and consequently, if that fulcrum be removed, the effort to move the lever AP about P will be the same as before.

whole collectively ; but, because it frequently happens, that the intire effect must be produced together, it is necessary to have recourse to mechanical contrivances, for accumulating and combining those separate and successive exertions of power. A man could with a single rope move in a certain time the same quantity of matter, which he could move in the same time by a machine, but he could do this only in detached parts. The machine enables him to move at once the whole mass, to which his unassisted strength would be utterly inadequate. There are six simple machines, or Mechanic Powers, the lever, the pulley, the inclined plane, the wheel and axle, the wedge, and the screw ; and the more complicated engines are only various combinations of these simple contrivances. Though the design of the Mechanic Powers is to communicate motion, the case of equilibrium is that which is investigated in the consideration of their properties, because, this being ascertained, an addition of force will overcome the resistance.

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* Philosophical Transactions, vol. 84. part 1.

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701 Levers are divided into three kinds, according to the various situation of the power and weight in regard to the fulcrum. The first kind is that, in which the power and weight are at contrary sides of the fulcrum; as in plate 4. fig. 8. *BC* represents a lever resting on the fulcrum *A*, *B* denotes the weight, and *C* the power. The second is that, in which the power and weight are at the same side of the fulcrum, but the power more remote than the weight; as in plate 5. fig. 2. *AB* represents a lever resting on the fulcrum *B*, *C* denotes the weight, and *A* the power. The third is that, in which the power and weight are at the same side of the fulcrum, but the weight more remote than the power; as in the preceding figure, if *C* denote the power, and *A* the weight.

It is obvious that in a lever of the first kind the fulcrum sustains the sum of the weight and power, but in each of the other kinds only the difference.

The three kinds of levers all agree in the same general relation of the power and weight. • In every lever, of whatsoever

• The power and weight are here supposed to act perpendicularly to the lever: in other cases they must be reciprocally proportional to perpendiculars drawn from the fulcrum to the directions in which they act. Let *BC* (plate 5. fig. 4.) be the lever supported at *A*; and let *BD* and *CH* represent the quantities, and directions of the forces acting at *B* and *C*. Draw *AM* and *AN* perpendicular to those directions; and draw *BF* and *CI* perpendicular to *BC*, and *DF*, *HI* parallel to it. Each of the forces *BD* and *CH* will thereby be resolved into two (and law of motion, cor. 4.) one of which, *DF* or *HI*, is parallel to the arm of the lever, and consequently cannot exert any pressure

whatsoever kind, that the power and weight may be in equilibrium, they must be reciprocally proportional to their distances from the fulcrum.

Let BC (plate 4. fig. 8.) be a lever of the first kind, supported by the fulcrum A . That the power at C and the weight at B may be in equilibrium, the former must be to the latter, as AB to AC .

If AB be conceived to be produced to X , so that BX may be equal to AC ; and CY , CZ be taken equal to AB ; the whole XY will be bisected in A : for $XB + BA = AC + CY$. Also $XB = BZ$, because it is equal to AC . Let the whole lever XY be a cylinder of uniform density, and, by the preceding lemma, the pressures of all its parts will produce the same effect as if they were united at the point A , the parts equidistant from A being considered together. The lever XY will therefore be in equilibrium. But the same must result, for the same reason, if the pressures of any two parts, as, XZ and ZY , into which it is divided, be conceived to be collected in the middle points of those parts. Since then XZ is bisected in B , let its whole

pressure upon it; and the other, BF or CI , is perpendicular to the arm to which it is applied. BF must therefore be to CI , as AC to AB . But, on account of the similar triangles BDE , BAM , $BD : BF :: AB : AM$; and it has been shewn that $BF : CI :: AC : AB$; and, on account of the similar triangles CHI , CAN , $CI : CH :: AN : AC$. Therefore, ex æquo perturbate, $BD : CH :: AN : AM$.

weight be supposed to be collected in B ; and, in the like manner, the whole weight of ZY to be collected in C ; and the equilibrium will continue. But in this case the weight at C would be to the weight at B , as ZY to ZX , or as $2 AB$ to $2 BZ$, or as $2 AB$ to $2 AC$, or as AB to AC .

The arms of levers of this kind are sometimes bent, so that they contain an angle, (plate 4. fig. 9.) and the fulcrum is placed at the angular point A . The power and weight must in all cases be conceived to be applied perpendicularly, as in the figure; or allowance must be made for the obliquity by the resolution of motion. Hence, in the motion of the lever itself an irregularity arises from the change of its position, on account of which it becomes oblique to the directions of the power and weight; and, although this proportionally affects both, it yet varies the intensity of the working of the machine, which in large engines is very inconvenient. This inconvenience is remedied, by annexing to the extremities of the lever small circular arches concentric to the fulcrum, or center of motion, and applying the power and weight by chains fastened to the upper extremities of the arches, as in plate 5. fig. 1. By this contrivance their actions are always exerted in the directions of tangents, and therefore perpendicularly to the distances from the fulcrum.

In demonstrating the property of the lever in the cases of those of the second and third kinds, they may be considered together; because they agree in this, that the

two



Fig. 1.

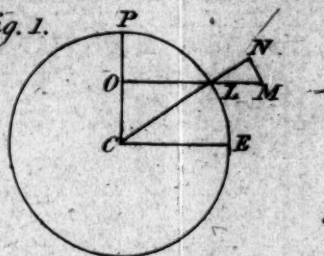


Fig. 2.

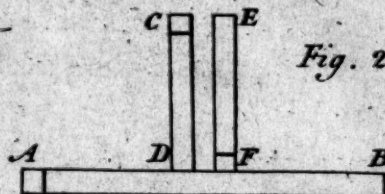


Fig. 3.



Fig. 4.

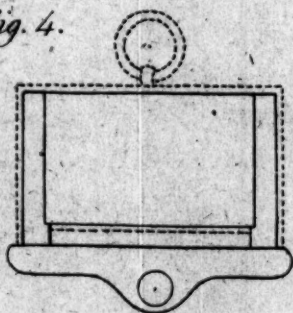


Fig. 5.

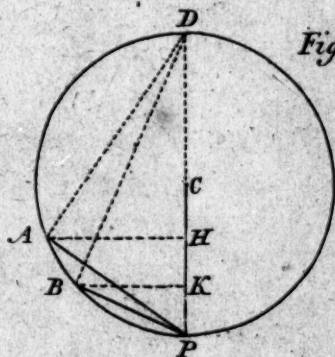


Fig. 6.



Fig. 7.



Fig. 8.

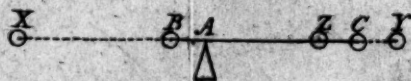
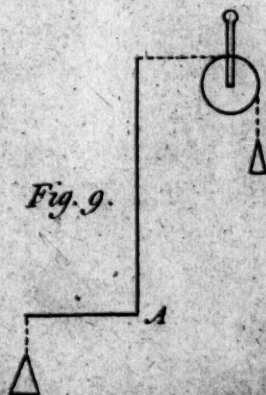


Fig. 9.



two forces, the power and weight, are applied at the same side of the center of motion; and in a general view it is indifferent which force shall be considered as the power, and which as the weight.

Let A and B (plate 5. fig. 2.) be two weights which balance each other upon the lever AB of the first kind, whose fulcrum is at C ; and, by the preceding case, $A : B :: BC : AC$. Let a power equal to the sum of the weights A and B be applied at C in a contrary direction, instead of the fulcrum. Since the pressure sustained by the fulcrum of a lever of the first kind is the sum of the weights, such a power would supply its place. The equilibrium would continue, and A or B might be considered as the fulcrum. But it has been already shewn, that $A : B :: BC : AC$. Therefore $A : C = A + B :: BC : AB$.

But though the same general relation of the power and weight is demonstrable of the three kinds of levers, they differ from each other in regard to the ratios which may subsist between the power and weight. In the first kind, since the fulcrum may be placed at any point between them, it is obvious that the power may have to the weight which it counterbalances any assignable ratio. In the second kind the power is always less, and in the third always greater than the weight. It should also be remarked, that, the power and weight being applied at given distances from the fulcrum, a lever of the second kind is more efficacious than one of the first; because in the second kind the power is to the weight, as the

the distance of the latter from the fulcrum to the whole length of the lever, but in the first kind only as the distance of the weight to the remaining part. The third kind does not give any assistance to the power, but is employed when the use of either of the others would be attended with inconvenience. In the theory the weight of the lever itself is neglected, and the lever is considered as an inflexible line destitute of weight. This weight however must produce some effect in practice, and in the first kind of levers it conspires with the power, but is exerted against it in the others.

To the first kind of levers may be partly reduced several sorts of instruments, such as scissars, pincers, and snuffers; each of which may be considered as composed of two levers, whose common fulcrum is the pin by which the parts are connected. A hammer used for drawing a nail is a bent lever of the same kind. To the second kind may in the like manner be reduced the oars and rudders of ships, a cutting-knife fixed at one end, and doors moving upon hinges. A ladder is reared against a wall agreeably to the operation of those of the third kind. It is remarkable that this last kind, though in general disadvantageous, should be employed in the construction of the human body. The arm, for instance, is a lever whose fulcrum is at the elbow, the weight is applied at the hand, and the power, or muscle, is applied at an intermediate point. The reason of this is however obviously the convenience of form, and the disadvantage is compensated by the additional strength of the muscle.

The

The investigation of the center of Gravity of a body, or of the common center of Gravity of two or more bodies, is an application of the theory of the level. This investigation is of the greatest importance in the practical application of mechanical principles. It discovers to us those points, in which the whole mass of a body, or of any number of bodies, may be considered as collected; and thereby enables us to apply to real cases those theoretical determinations, which had treated of bodies merely as points. The center of Gravity of a body, or the common center of two or more bodies, is accordingly defined to be that point, in which the weight of the parts of the body, or of the different bodies, may be considered as united.

* That there is such a point in every body, may be thus shewn. Let A, B, C, D , &c. (plate 5. fig. 3.) be any number of particles of matter, composing a body. Suppose A and B to be connected by AB , an inflexible line without weight, and let AB be divided in E , so that $A : B :: BE : EA$. Then will A and B balance each other upon E , or if E be supported, A and B will be supported in all positions. Also the pressure upon E will be the sum of the weights A and B . Join EC , and let $CF : FE :: A + B : C$. Then, if F be supported, E and C will be supported, that is, A, B and C will be supported; and the pressure upon F will be the sum of

* Two principles not strictly true are assumed in this argument; one that the weights of the particles act in parallel directions, the other that these weights are not affected by the difference of the distances from the center of the earth.

the weights *A*, *B* and *C*. And so with any number of particles. The last resulting point will be the center of Gravity of all these particles, or of the body which they compose.

Although in comparing these several points, the directions of the weight are often oblique to the lines, by which they are supposed to be joined, yet, these directions being all parallel, the obliquities in regard to any of these lines are similar, and the same principle of equilibrium will still hold.

From what has been premised it appears, that if a body be supported by its center of Gravity, it will continue in any position in which it is placed. The weights of its parts being in equilibrium about that point, no force can be exerted by them to change their position. On the contrary, the whole weight of the body being collected in that point, if it be not supported, it will, in obedience to the operation of Gravity, endeavour to descend to the center of the earth; and the body, if freely suspended, will not rest unless the center of Gravity be perpendicularly beneath the point of suspension. A body therefore, which is supported by any point different from its center of Gravity, can rest only in two positions: 1st when the center of Gravity is perpendicularly above the point of suspension, in which case it rests upon it; and 2^{ly} when it is perpendicularly beneath that point, in which case it is again sustained by it.

Various methods have been proposed for discovering the

the center of Gravity of an homogeneous and regular body; but the only general determination of this center is derived from the property mentioned in the preceding paragraph, its tendency towards the center of the earth. If a body be suspended successively by different points, and right lines be drawn through the points of suspension perpendicular to the horizon, the center of Gravity will lie in each of these perpendiculars, and consequently in the point of their intersection.

The consideration of the center of Gravity enables us to determine in what case a body will fall, and when it will be supported. It has been shewn, that the center of Gravity of a body endeavours to descend towards the center of the earth, and consequently in a line perpendicular to the horizon. If this line, which is called its *line of direction*, fall within the base of the body, the center of Gravity will be supported by the body, and the body will consequently retain its position. But if the line of direction fall without the base, it will no longer receive support, and the body will fall. Hence it follows, that of two portions of different kinds of matter, resting on equal bases, and containing equal weights, that which is more bulky, and therefore more lofty, will be more easily overturned. The body being more lofty, its center of Gravity will also be more elevated, and consequently its line of direction will be more easily thrown without the base.

The same principle will also enable us to explain the different methods, in which bodies of various forms de-

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scend

ascend along the same inclined plane. Some, as cubes, slide down an inclined plane, whilst others, as globes or cylinders, roll. The cause of this difference is, that the lines of direction of the centers of Gravity of the cubes will, unless the plane be too much elevated, continue to fall within their bases; whilst those of the globes or cylinders must, in any elevation, be thrown without those parts in which they rest upon the plane. It must however be observed, that no body would roll down an inclined plane if the friction were wholly removed. If the friction of the plane did not detain the lower parts of the body, no rotation could be produced by the tendency of the center of Gravity, and a globe or cylinder would slide like a cube.

If *A, B, C, D, &c.* (plate 5. fig. 3.) represent the centers of Gravity of so many distinct bodies, each body being conceived to be collected into its center of Gravity, *E* will be the common center of *A* and *B*; *F* the common center of *A, B* and *C*; *G* the common center of *A, B, C* and *D*; and so with any number.

A balance is a lever of the first kind, employed for comparing the weights of bodies, and consequently (schol. 1. to the first law of Gravity) the quantities of matter which they contain.

There are two kinds of balances, the common balance, and the *statera Romana* or steel-yard.

With regard to each of these it must be observed, that
a weight,

a weight, whatever be the length of the cord by which it hangs, acts in the same manner as if its center of Gravity were applied at the point of suspension. This is confirmed by the following experiment. Let a weight appended at one arm of a balance be counterpoised by a weight at the other, and let it, by means of a cord, be hung at different distances below the point of suspension. The position of the balance will remain unvaried, and the weights will consequently continue to counterpoise each other. Hence it follows, that the distance of the weight from the point of suspension does not affect its operation upon the arm of the balance,

The common balance is a lever whose arms are equal, and which will consequently continue in equilibrium when equal weights are applied at the extremities of the arms. If the axis of motion passed through the center of Gravity of the beam, its quiescence in any position would indicate the equality of the weights applied to the arms, because it would indicate that the center of Gravity had not been changed by loading the beam. It has however been found more convenient, that some fixed position of the beam should denote the equality of the weights, and none is so eligible as the horizontal. That the beam may have this position, when its arms are in equilibrium, it is necessary that a right line passing through its center of Gravity and center of motion should be perpendicular to its length. For, since the beam will not rest unless the center of Gravity be perpendicularly above or beneath the center of motion, it is obvious that it cannot rest unless the length of the beam

be parallel to the horizon. This horizontal position is ascertained by an index, which is a continuation of the right line passing through the centers of Gravity and motion. The center of motion is taken above the center of Gravity of the beam, because in this case the center of Gravity will bring the beam to its true position: whereas, if the center of motion were taken beneath the center of Gravity of the beam, it would be necessary to adjust the beam to its horizontal position; and any error would leave the center of Gravity at liberty to descend, and disturb the operation.

By the length of the balance is meant the interval between the points of suspension; and these points should be so taken, that the center of Gravity of the beam may lie between them. Since the weights act as if they were applied at the points of suspension, the addition of equal weights will not in this case alter the center of Gravity of the beam, and consequently will not disturb its horizontal position. The vibrations of a balance are quicker, and the horizontal tendency stronger, the greater is the interval between the center of motion and the center of Gravity. To lessen the inaccuracies which might be occasioned by friction, the balance is made to turn on an edge, and the weights are also suspended on edges. For the same purpose, it has been recommended, that the arms should be made long; since any inequality of the weights, thus applied at the extremity of a longer lever, will more easily overcome the friction. This must however be regulated by the strength of the beam. If the arms should be liable to be bent by the weights,

weights, the comparison of the weights will be inaccurate.

If the arms be of unequal lengths, whilst no allowance is made for this inequality, it is a false or deceitful balance; because unequal weights will counterpoise each other. This appears from the property of the lever. Those weights will balance each other, which are in the reciprocal ratio of their distances from the center of motion. Since therefore these distances are unequal, the counterpoising weights must also be unequal.

It is observable, that if any substance be weighed successively in the scales suspended from both arms of a false balance, the true weight will be a geometrical mean between the two false weights. If it be suspended from the shorter arm, its counterpoise will be to it in the ratio of the shorter arm to the longer; and if it be suspended from the longer, it will be to its counterpoise in the same ratio. It is therefore a geometrical mean between them. The true weight may consequently be found very nearly, by taking an arithmetical mean between the two counterpoises. The proportional weights of various bodies may indeed be accurately determined by any balance however false, if they are always placed in the same scale; for, since the ratio of the arms is invariable, the weights of the bodies will have all the same ratio to the counterpoising weights.

The *statera Romana*, or steel-yard, is a lever of the first kind, whose arms, are unequal. These arms how-

ever

ever are so constructed, as to be in equilibrium; and the body, whose weight is sought, is suspended from the shorter arm, and its weight is estimated by the distance of a sliding weight on the other arm from the center of motion. Thus, if the sliding weight be, when the equilibrium takes place, at a point which is distant from the center of motion by an interval equal to the length of the shorter arm, the weight of the body is equal to that weight; if it be distant by twice that interval, the weight of the body is double of that weight; and so in other cases. The principle of this is the same as that of the common balance; namely, that in a case of equilibrium the weights must be reciprocally proportional to the distances of the points of suspension from the center of motion. The difference between them consists only in this: that in the common balance the distances are unvaried, and the weights of bodies are estimated by comparing them with different counterpoising weights, to which they must consequently be equal; whereas in the steel-yard the counterpoising weight is unvaried, and the weights of bodies are estimated by comparing the distances at which it is necessary to place that weight, that it may be in equilibrium with the bodies.

By the property of the lever we are also enabled to determine the pressures of beams supporting weights,

If a beam supporting a weight, and resting on two props or fulcrums, be parallel to the horizon, the sum of the pressures upon the fulcrums will be equal to the weight;

weight; and the pressure sustained by each fulcrum will be to that sustained by the other, in the reciprocal ratio of their distances from the weight. Let AB (plate 5. fig. 2.) be a beam supported at A and B , and sustaining a weight at C . The pressure at A or B must be equal to that power, which would be just sufficient to sustain the beam, if applied at the same point; and the beam may be considered as a lever of the second kind, resting on a fulcrum at A or B , and supported by a power at the other extremity. If then A be considered as the fulcrum, the power at B must be to the weight, as AC to AB ; and, if B be considered as the fulcrum, the weight must be to the power at A , as AB to CB . Therefore the power necessary for supporting the beam at B is to the power necessary for supporting the beam at A , that is, the pressure at B is to the pressure at A , as AC to BC .

If the beam be inclined to the horizon, the determination of the pressures will be * various, according to the various directions in which the support given by the fulcrums is exerted. Thus, the beam may at its upper extremity rest against an inclined or vertical plane, and against an horizontal one at the other; or its upper extremity may be sustained at a fixed point, and its lower rest against an inclined plane; or it may rest upon two oblique supports. In every case however the pressures must be such as, if compounded according to their respective directions, will counterbalance the weight of

* Emerson's Principles of Mechanics, sect. 7.

the body. Hence, since in every composition of oblique motions there is a loss of force, (see page 10), the sum of the pressures upon the fulcrums will in these cases be greater than the weight supported.

The weight of the beam itself must be considered as a part of the weight supported, and referred to its own center of Gravity. The common center of Gravity of the beam and weight must determine the place of the weight.

* The strength of a beam may be estimated according to the principle of a lever. Let AB (plate 5. fig. 5.) be a beam, placed horizontally, and fixed at the extremity B ; and let any force be applied perpendicularly at P , to break the beam. Conceive the beam to be divided into an indefinite number of very thin beams by planes parallel to its upper and lower surfaces. To break the beam, the power P must overcome the cohesion of the parts of these thin beams in some line EF ; and the force is accordingly exerted upon so many bent levers, which have all the same longer arm AE , and whose shorter arms are the distances of the several thin beams from the point E .

Hence appears the reason, why the rafters of an house are stronger when placed on their edges, than when flat; since the strength of a beam must depend more upon its depth, than upon its breadth. By an in-

* Ibid. sect. 8.

crease of breadth, the resistance of each of the indefinitely thin beams is strengthened in the same proportion; but an increase of depth augments in the same proportion, not only the number of these beams, and consequently the resistance of the whole, but also the efficacy of that resistance by increasing its distance from the angle of the lever.

A pulley is a small wheel moveable about its center, in the circumference of which a groove is formed to admit a rope or chain.

Pulleys are used in Mechanics in two different ways. They are either fixed by their centers, so as not to change their place during the working of the machine; or moveable.

In a single fixed pulley (plate 5. fig. 6.), that there may be an equilibrium, the power must be equal to the weight.

For in this case the two parts of the rope, by which the power and weight are sustained, must be equally stretched; because otherwise a motion would take place. Therefore the weights sustained, or the power and weight, must be equal.

* In a single moveable pulley (plate 5. fig. 7.) that there may be an equilibrium, the power must be equal to one

* In this and the following cases the ropes are supposed to be parallel. See Wood's Principles of Mechanics, sect. 4.

half of the weight, that of the moveable pulley itself being considered as a part of the weight.

Let a rope fixed at *E* pass under the moveable pulley *A*, and over the fixed pulley *B*. Let the weight be annexed to the center of the pulley *A*, and the power be applied at *B*. Then since the two parts of the rope, *EA* and *AB*, sustain the weight, and are equally stretched, each of them sustains the half of the weight. But, by the preceding case, the rope *BP* sustains the same weight, which *AB* sustains. The power is consequently equal to the half of the weight. The moveable pulley therefore assists the efficacy of the power; but the fixed pulley is only serviceable in changing the direction in which that power is exerted, or in stretching the ropes which pass round a number of moveable pulleys.

In a system of fixed and moveable pulleys in which the same rope passes round all the pulleys, as in plate 5. fig. 8. and 9, that there may be an equilibrium, the power must be to the weight, as 1 to the number of the parts of the rope at the lower block.

For, since all these parts of the rope are equally stretched, each will sustain a part corresponding to their number; and the part, to which the power is applied sustains an equal pressure, because the equilibrium of the machine is supposed. The power is therefore such a part of the weight, as corresponds to the number of the

the parts of the rope at the lower block; that is, the power is to the weight, as 1 to that number.

If the extremity of the rope be fastened above, as in fig. 8, this ratio is that of 1 to twice the number of moveable pulleys. If it be fastened to the lower block, as in fig. 9, it is the ratio of 1 to twice the number of moveable pulleys increased by 1. In this and the following cases the weight of all the moveable pulleys must be considered as a part of the weight sustained.

In a system of one fixed and any number of moveable pulleys, in which a separate rope fastened above passes round each moveable pulley, as in plate 6. fig. 1, that there may be an equilibrium, the power must be to the weight, as 1 to the last term of a duple progression, whose first term is 1, and number of terms the same with the whole number of pulleys.

For the power is to the weight at *B*, as 1 to 2; and the weight at *B* is to the weight at *C*, as 2 to 4; and the weight at *C* is to the weight at *D*, as 4 to 8; and so with any number of moveable pulleys. In this progression the second term corresponds to the weight at the first moveable pulley *B*; and therefore the number of terms is the same with the whole number of pulleys.

In that system of one fixed and any number of moveable pulleys, in which a separate rope fastened to the weight passes round each pulley, as in plate 6. fig. 2, that there may be an equilibrium, the power must be to the weight,

as 1 to the sum of the terms of a duple progression, whose first term is 1, and number of terms the same as the number of pulleys.

For the weight sustained by the rope *DA*, is equal to the power *P*. Therefore the weight sustained by *BA*, and consequently that sustained by *BE*, is equal to $2P$. In the same manner the rope *CF* sustains $4P$; and so with any number of pulleys.

* A given power applied to this system of pulleys will sustain a greater weight, than when the same power is applied to any equal number of pulleys disposed according to any other system.

A considerable improvement in the construction of pulleys has been invented by Mr. James White, who obtained a patent for his contrivance. It is designed to remedy the lateral friction and shaking motion of that system of pulleys, in which a single rope passes round all the pulleys. When such a system is in motion, the pulleys must turn with various velocities; because each must turn with the velocity of that part of the rope which is passing round it. It is evident therefore that, whilst the weight ascends, the pulley most remote from the power moves with a velocity corresponding to the shortening of one part of the rope; the next pulley with a velocity corresponding to the shortening of two parts of the rope; and so in the rest. The velocities of the pulleys conse-

* Atwood's Analysis of a Course of Lectures, Art. 30.

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Fig. 1.

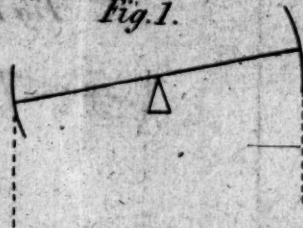


Fig 2.

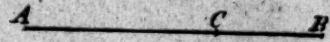


Fig. 3.

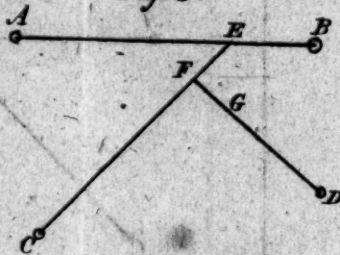


Fig. 5.

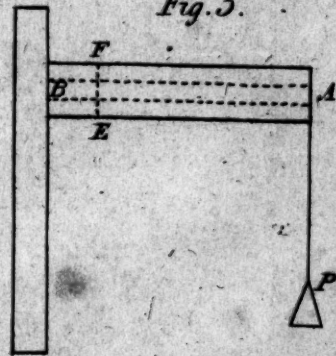


Fig. 4.

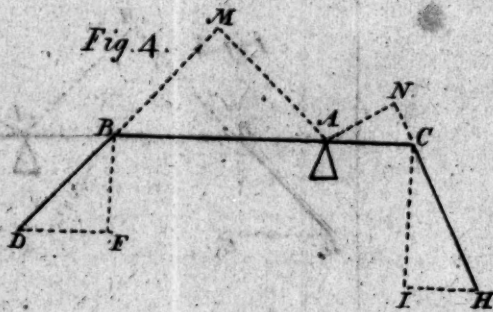


Fig. 6.



Fig. 7.

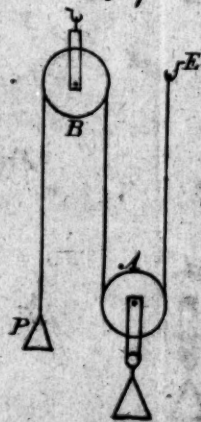


Fig. 8.

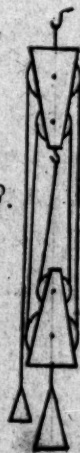
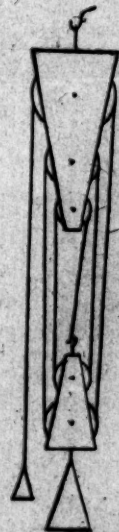


Fig. 9.



quently increase in an arithmetical progression, whose first term and common difference are 1. Mr. White's contrivance consists in causing all the pulleys of the same block to turn together. This he effected by diminishing the diameters of all the pulleys, except that nearest to the power, in the same proportions in which their velocities had been diminished in the other constructions. For since the peripheries of the pulleys are diminished in the same proportions in which their diameters are diminished, that is, in the proportion of their velocities of rotation, the times of rotation will be equal (see page 18.). He was accordingly enabled to form each block of one piece (plate 6. fig. 3.) with grooves corresponding to the several distinct pulleys.

Some writers have contended that the pulley may be explained by reducing it to the lever; and have accordingly considered the fixed pulley as operating according to the principle of a lever of the first, and the moveable pulley as acting like a lever of the second kind. The lever in each case they conceived to be a diameter of the pulley connecting the contacts of the rope; and in the former they considered the lever as supported at the center, in the latter as supported at the contact more remote from the power. But Bishop Hamilton has shewn the fallacy of this solution. It depends intirely upon a consideration of the magnitude of the wheels, which do not at all contribute to the support of the weights. If the ropes were passed round mathematical points, their tension would be just the same, and the same powers would sustain the same weights. The wheels are used merely

merely to diminish the friction in working the machine, agreeably to a principle which will be explained in treating of the Mechanic Power called the Wheel and Axle.

The inclined plane is reckoned among the Mechanic Powers, because a less force may support a body which rests upon an inclined plane, than that which would be required for supporting it if it hung freely; and consequently a less force may draw it up the plane, than that which would be necessary for raising it perpendicularly.

* If a weight be sustained on an inclined plane by a power which acts in a direction parallel to the plane, the power will be to the weight, as the height of the plane to its length.

Let AB (plate 6. fig. 4.) be the inclined plane, and the body at P be sustained by a power acting in the direction PA . The body is kept at rest by three forces, the power acting in the direction PA , the force of Gravity which is perpendicular to the horizon, and the reac-

* The general principle, whatever may be the direction of the power, is that the power must be to the weight, as the sine of the elevation of the plane is to the cosine of the angle in which the direction of the power is inclined to the plane.

Thus, let the body P (plate 6. fig. 5.) be sustained by a power acting in the direction PV , and the three forces will be proportional to the sides of the triangle VPC , and the power to the weight, as PV to VC , and consequently as the sine of the angle PCV to the sine of the angle VPC or VPE ; that is, on account of the similar triangles APC and ABC , as the sine of the angle ABC , to the sine of the angle VPE , or the cosine of the angle VPA .

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tion of the plane which (3rd. law of motion) is contrary to the pressure against the plane, and is therefore perpendicular to the plane. These three forces must consequently (2nd. law of motion cor. 3.) be proportional to the sides of any triangle drawn parallel to their respective directions. Let PD be drawn perpendicular to AB , and DF perpendicular to BC . Then will the triangle PDF have its sides respectively parallel to the directions of the forces. Therefore the sustaining power is to the weight, as PF is to FD , or (Eucl. book 6. prop. 8. cor.) as FD to FB , or, on account of similar triangles, as AC to AB .

The wheel and axle consists of an axle united to a wheel, so as to turn with it. The power is applied at the periphery of the wheel, and the weight is applied to the axle, commonly by a rope which winds about it.

In this machine, that there may be an equilibrium, the power must be to the weight as the radius of the axle to the radius of the wheel.

The same power is required for supporting the weight whatever be the point of the axle to which the wheel is applied. Let the wheel therefore be supposed to be so placed, that the power and weight may act in the same plane (plate 6. fig. 6.); and let CA , CB be those radii of the wheel and axle, at whose extremities the power and weight act. Then the machine becomes a lever of the first kind ACB , whose fulcrum is at C ; and the power is to the weight, as CB to CA . When the pow-

er acts at the extremity of any oblique radius, as *CD*, the machine acts like a bent lever; and when the power acts at *E*, the machine becomes a lever of the second kind: but the same ratio of the power and weight remains. Sometimes, instead of the wheel, a lever is inserted into the axle, and the power applied at its extremity.

If the power or weight be applied by a rope, this ratio will be somewhat varied, because the radius of the wheel or axle must be increased by the addition of the half of the thickness of the rope every time that it is twisted round it.

Two or more wheels and axles are usually combined by the means of teeth placed in their peripheries. The teeth of the first axle being applied to those of the second wheel, motion is communicated from the former to the latter; and so with any greater number. * In such a combination, that there may be an equilibrium, the power must be to the weight, in the ratio compounded of the several ratios, which the power should have to the weight with each wheel and axle separately employed; that is, in the ratio compounded of the several ratios, which the radii of the axles have to the radii of their respective wheels, or as the product of the former radii to that of the latter. And because the teeth respectively applied to each other must be of equal magnitudes, the numbers of teeth in the wheels and axles respectively applied to each other

* Parkinson's System of Mechanics, chap. 8.

are proportional to their peripheries, and consequently to their radii. The power must therefore be to the weight in the ratio compounded of the ratio of the radius of the last axle to the radius of the first wheel, and of the several ratios of the numbers of teeth in the axles to the numbers of teeth in the wheels. The teeth placed in the periphery of an axle form what is called a Pinion.

If an axle be so formed with a spiral groove, that the rope or chain may be successively passed over parts of various thicknesses, a varying weight or power applied to the axle may be so managed, as to produce an uniform motion of the wheel. For the efficacy of this power in turning the wheel must be reciprocally proportional to the thickness of that part of the axle to which it is applied, since the weight or resistance and the radius of the wheel are unvaried.

If therefore the power, when strongest, be applied at the thinnest part of the axle; and then successively at parts increasing in thickness in the same proportion in which it diminishes in intensity; the effect will be uniform. This is the case of a watch. The spring, included in the spring-box, is exerted most powerfully at the beginning; and its power is continually diminished until the watch stops, or is again wound up: and accordingly the chain is at the beginning applied to the thinnest part of the axle about which it is wound; and afterwards to parts continually increasing.

It must be observed, that in watches and clocks the

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application

application of the wheel and axle is inverted, the power being applied to the axle for producing the motion of the wheel.

One of the greatest inconveniencies in the construction of machines is the friction of their parts. The surfaces of all bodies whatever are in some degree rough; and when the surface of one body is moved along that of the other, the prominent parts of the former will fall into the cavities of the latter, and cause a resistance. This can only be overcome by a force that shall raise the prominent parts of the former over those of the latter. The whole of the moving body must therefore be raised to an height equal to that of the prominencies over which it is moved; and consequently the force must sustain a part of the weight of the moving body. It is not necessary that it should sustain the whole weight, because the motion is of the same nature with that which is performed up an inclined plane.

Friction may be in some degree diminished by diminishing the roughness of the surfaces; but notwithstanding this precaution it will still remain considerable. The most effectual method of reducing it is the application of wheels, which act according to the principle of the wheel and axle. For explaining the operation of wheels in diminishing friction, it must be observed that, whilst the weight of the moving body and the roughness of the surfaces remain the same, the friction is not considerably varied by any variation in the magnitude of the contiguous parts. This is ascertained by experiments, which
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shew that the same weight will move along the same surface different pieces of the same polished substance, as box, which are loaded with equal weights, but rest on unequal bases. Those wheels therefore, by which the motion of carriages is facilitated, do not effect this by resting on fewer points of contact. The advantage of using them arises principally from the diminution of the velocity, with which the moving surface is applied to the fixed one; on which account the force, or momentum, of the friction is diminished. If the carriage had not wheels, or if the rotation of the wheels were stopped, as when a drag is put on a wheel, the velocity with which the moving surface is applied to the fixed one would be the velocity of the carriage itself; but since the periphery of each wheel yields to the roughness of the surface on which it moves, the friction is transferred to the axle, and the velocity is now only that with which the nave of the wheel is applied to the axle. The effect of the friction is therefore diminished, agreeably to the operation of the Mechanic Power, in the ratio of the radius of the wheel to the radius of the axle.

From this it follows, that the larger the wheels of a carriage are made, the more effectually is the resistance of friction overcome. Large wheels have also other advantages. On account of the smallness of their curvatures, they sink less between two adjacent prominencies of the surface upon which the carriage moves, and consequently require less force for raising them over those prominencies; and for the same reason they operate more powerfully in depressing such obstacles as will yield to the

weight of the carriage. Two other considerations however set limits to the magnitude of wheels. One of these is the convenience of turning the carriage, which may be done with more ease and safety when the wheels are small. The other is the direction of the power by which the carriage is drawn. The force of an horse drawing a carriage is exerted at his breast; and therefore, if the axle were higher than his breast, a part of his force would be employed in drawing the carriage towards the earth, and thereby increasing the load. It is on the contrary convenient that the axle should be lower than the breast of the horse, since in this case a part of his force is exerted in raising the carriage over the obstacles of the surface. Mr. Edgworth has remarked an obstruction to the motion of carriages, which had not been observed before. This is their *vis inertiae*. "When a carriage," * says he, "has been once set in motion upon a smooth road with any given velocity, its motion, so long as that velocity is continued, is neither retarded nor promoted by its *vis inertiae*; but whenever it passes over any height, not only the weight of the carriage must be lifted up, but the *vis inertiae* of that weight must be overcome in a new direction, and as much velocity must be communicated to it in that new direction, as will enable it to rise to the height of the obstacle whilst it passes over its base." This is in a very considerable degree obviated, as he has remarked, by the application of springs, which render the motion of the carriage less abrupt. It is probably on the same account, that ships sail faster

* Transactions of the Royal Irish Academy for 1788.

when their rigging is slackened to a certain degree, so as to become more elastic than in its ordinary state.

To diminish still farther the resistance of the friction, the axle has been made to rest on the peripheries of other wheels, as in the machine represented in plate 1. fig. 1. The operation of these, which have been peculiarly denominated Friction-wheels, is similar to that which has been already described. The peripheries of these wheels moving with the axle which rest upon them, the friction is again transferred to their axes, and consequently again diminished in the ratio of the radii of these wheels to the radii of their axes.

A wedge is a hard body of a triangular prismatic figure. The power is applied to the quadrilateral base of the wedge, generally by impulse; and the resistance is the cohesion of the substance into which the wedge is driven.

When the resistance acts in a direction perpendicular to each side, that there may be an equilibrium, the power must be to the resistance, as the length of the base of the wedge to the sum of the lengths of the sides, or as the sine of the semi-angle at the vertex of the wedge to radius.*

Let

* The general principle, whatever be the direction of the resistances, is that the power must be to the resistances, as the rectangle under the sines of half the vertical angle of the wedge and the inclination of the resistances to the sides to the square of the radius.

Let

Let AVB (plate 6. fig. 7.) be a triangular section of a wedge, and let the lines CE , ce express the directions and quantities of the resistances exerted against the sides. Let these be resolved into EF , eF parallel to the base, and CF , cF perpendicular to it (2nd law of motion cor. 4.). The forces represented by EF , eF , being equal and opposite, destroy each other. There remains a force expressed by $2CF$; and consequently there will be an equilibrium, if the power applied at C in the contrary direction be equivalent to this force. The power is therefore in this case to the resistance, as $2FC$ to $EC + cC$, or as FC to EC , and consequently as EC to CV (Eucl. book 6. prop. 8. cor.) or as AC the sine of the semi-angle AVC to AV the radius; and therefore also as the base AB to the sum of the sides AV , BV .

Since the power requisite for overcoming a given resistance, is to that resistance, as the sine of the semi-angle at the vertex of the wedge to radius; the powers necessary with different wedges must be proportional to the

Let the directions and quantities of the resistances be CD and Cd (plate 6. fig. 7.). Resolve them into DE and de in the directions of the sides, and CE , Ce perpendicular to them. The former cannot produce any effect. Resolve then the latter CE , Ce into EF , eF parallel to the base, and FC , fC perpendicular to it. EF , eF destroy each other; and the power will, as in the particular case, be $2CF$. But $FC : EC ::$ sine of the angle FEC or CVA : radius; and $EC : DC ::$ sine of the angle EDC : radius. Therefore $FC : DC ::$ sine of the angle $CVA \times$ sine of the angle EDC : the square of radius.

For a view of the difficulties which have embarrassed the consideration of this Mechanic Power, see Parkinson's System of Mechanics, Art. 391.

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sines of their semi-angles. A wedge is said to be more efficacious than another, if it require a less power for overcoming a given resistance. The efficacies of wedges are therefore reciprocally proportional to the powers which they require, and consequently are reciprocally proportional to the sines of their semi-angles.

The wedge is used for dividing the parts of hard substances; and all sharp instruments are reducible to it. In theory, the property of the wedge has been applied in a very simple and elegant manner by * Bishop Young, to determine the equilibration of arches, a problem of great importance in architecture. The arch-stones he considers as so many wedges, urged by the incumbent weight, and endeavouring to split the arch.

The screw consists of two parts, which have been denominated the Male and Female Screw. The former is a solid cylinder, so cut as to have a prominent part carried round it in a spiral form. The latter is an hollow cylinder, with a spiral groove corresponding to the spiral prominence of the former. The two screws being thus exactly adapted to each other, the solid or hollow cylinder, as the case requires, may be moved round the common axis, by a bar perpendicular to that axis; and a motion will be produced in the direction of the axis, by means of the spiral thread.

The screw is not in reality a simple Mechanic Power,

* Transactions of the Royal Irish Academy for the year 1789.

but

but a combination of the inclined plane with the wheel and axle. If a plane triangle ABC (plate 6. fig. 8.) whose base BC is equal to the circumference of a solid cylinder be applied to it in such a manner, that its base BC may coincide with that of the cylinder, BA will form a spiral on its surface; and by applying successively to the cylinder triangles similar and equal to ABC , so that their bases BC may form parallel circles, the spiral may be continued. Hence it appears, that the spiral acts like an inclined plane AB ; and the bar, by which the screw is turned, has the effect of the wheel and axle.

For determining the power necessary to produce an equilibrium with the screw, the following property of the inclined plane must be premised.

If the power, by which a body is sustained on an inclined plane, act in a direction parallel to the base, it must be to the weight as the height to the base.

Let AB (plate 6. fig. 9.) be the inclined plane, and P the place of the body. From P draw PC perpendicular to the plane, and PD perpendicular to the base; and, since the three forces acting on P must be proportional to the sides of the triangle PDC , to which their directions are respectively parallel (2nd. law of motion cor. 3.), the power must be to the weight, as DC to PD , or (Eucl. book. 6. prop. 8. cor.) as PD : DB , or as AC to CB .

To produce an equilibrium with the screw, the power must be to the resistance, as the distance between two contiguous

contiguous threads, measured in the direction of the axis, to the periphery described by the power.

Let BAD , (plate 6. fig. 10.) represent a section of the screw made by a plane perpendicular to its axis, and the periphery BAD will be the base of one of those triangles, by the application of which the spiral thread is conceived to be generated. Let the power be conceived to be applied at B , in the direction of the base of the spiral; and let the resistance, which acts in a direction parallel to the axis of the screw, and consequently perpendicular to the base of the spiral, be conceived to be united in one point of the spiral. Then, by the preceding property of the inclined plane, the power applied at B must be to the resistance, as the height of the spiral to its base, that is, as the distance between two contiguous threads, measured in the direction of the axis, to the periphery BAD . But the effect of a power applied at P will be equivalent to the former applied at B , if it be to it, as the periphery BAD is to that described by the power at P . Therefore the necessary power at B is to the resistance, as the distance of the threads to the periphery BAD ; and the necessary power at P is to that at B , as the periphery BAD to the periphery described by P . Consequently, *ex æquo perturbate*, the power at P is to the resistance, as the distance of the threads to the periphery described by P .

It has been remarked, that the screw is not strictly a simple Mechanic Power: and it may be farther observed, that there are indeed only two, which can truly be so de-

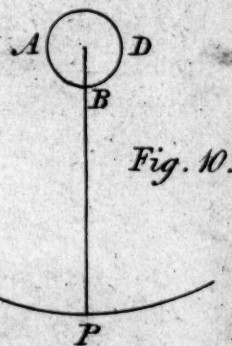
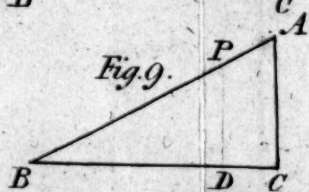
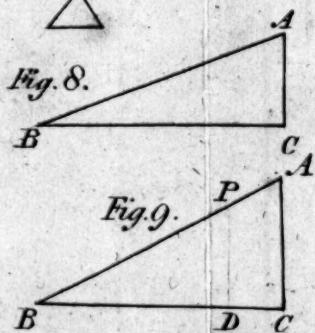
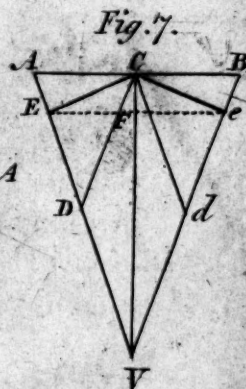
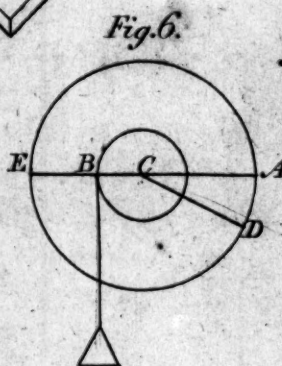
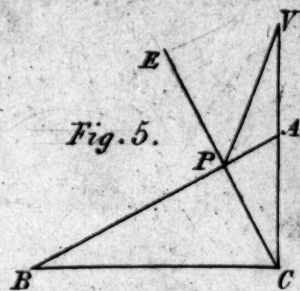
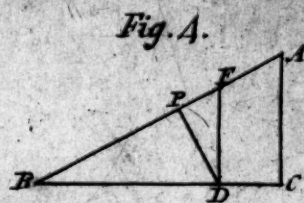
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nominated; the lever and the pulley. The wheel and axle has already been reduced to the former of these; and the inclined plane may also be considered as of the same class. These two, as has been mentioned, compose the screw. Lastly, the wedge has been conceived to be composed of two inclined planes joined at a common base, the resistance being in this case the weight to be sustained.

Experiments made with the screw differ more from the theory, than those which are made with any other Mechanic Power; because the friction bears a greater proportion to the other resistance. It has been already observed, that, the weight been given, the friction varies nearly as the velocity with which one surface is moved over another. In the screw this velocity exceeds that with which the screw itself is driven forward, because it is the velocity with which the parts of the thread are applied to the spiral groove, whilst the screw is driven forward only through a space equal to the corresponding segment of its axis. The friction must consequently bear a great proportion to the resistance. This is however attended with advantage. The principal use of the screw being to compress bodies together, its great friction is serviceable in preserving its position without requiring a continued exertion of the power.

A very ingenious and useful application of the screw has been contrived, for the purpose of subdividing small spaces. It is in this case called a Micrometer Screw, and is adapted to astronomical instruments. The number
of



of turns which the screw must make in advancing an inch, is known by observing the number of threads contained within that length of the axis. If it must make fifty turns, each turn will ascertain the fiftieth part of an inch. Besides this, on the extremity of the screw is fixed a circular plate, whose edge is divided into a certain number of equal parts. If the edge be divided into an hundred equal parts, each of these will indicate the hundredth part of a turn, that is, the hundredth part of the fiftieth part of an inch, or the five thousandth part of an inch.*

The male screw is sometimes used only for the communication of motion from one part of a compound machine to another, and is then called a Perpetual Screw. It is for this purpose so fixed, that it can only turn round its axis without any progressive motion; and serves merely to turn a wheel, by receiving its teeth into the intervals between the threads. Such a screw differs from the axle in the wheel in this, that the spiral thread is substituted for the teeth or pinion of the former, and that it changes the direction of the motion which it communicates. Since each revolution of the screw turns one tooth of the wheel applied to it, the efficacy of the machine is multiplied by the number of teeth in that wheel.

* This use of the screw having rendered the accurate construction of it very important, much attention has been given to the mechanical process, of which an account is contained in Nicholson's Journal of Nat. Phil. &c. for July 1797.

It has been observed, that whatever is gained in power by the use of any machine, is lost in time. The truth of this appears from comparing the velocities of the power and weight in the several Mechanic Powers; for it is found that the ratio of these velocities is the reciprocal of that of the power and weight when in equilibrium. * Thus, the velocities of the power and weight applied to a lever are proportional to the small arches which they describe in the same time, and therefore, since these arches subtend the same or equal angles at the center of motion, proportional also to the distances from the fulcrum: but it has been shewn, that the power and weight which balance each other are reciprocally proportional to those distances. In a single moveable pulley, or any combination of pulleys, the velocity of the weight is estimated by comparing the space through which it is moved, with the shortenings of the several parts of the rope; for the same quantities by which those ropes are shortened, to which the weight is applied, are added to that which sustains the power, and therefore determine its velocity. In the inclined plane the oblique ascent of the weight along the plane or the velocity of the power, is to its perpendicular ascent, as the length of the plane to its height; or in the reciprocal ratio of the power and weight. In the wheel and axle the velocities of the power and weight are in the ratio of the periphery of the wheel to that of the axle; and consequently as their radii. For similar reasons, in the screw the velocity of

* For the cases of powers obliquely applied, see Wood's Principles of Mechanic's Art. 149.

the power is to that of the resistance as the periphery described by the power to the distance between two contiguous threads measured in the direction of the axis; and in the wedge the velocity of the power is to that of the resistance as CV to CE (plate 6. fig. 7.) or on account of similar triangles, as AV to AC .

When a force continues to operate on a machine, as a weight or the impulse of water, the machine would be continually accelerated, if the friction of its parts and the resistance of the air, which increase with its velocity, did not limit the acceleration. But as the intensity of the moving force, or of these resistances, varies, the motion of the machine is subject to variation. To remedy this inconvenience, fliers of various kinds have been invented. These are commonly heavy wheels, connected with those parts of machines which move with the greatest velocities; and they act by their inertia. If the moving force be suddenly increased, a considerable part of the augmentation will be employed in communicating additional motion gradually to the flier; and if it should be suddenly diminished, the momentum of the flier will resist a sudden diminution of the motion of the machine. The pendulum of a clock and the balance of a watch are fliers of another kind, since they are actuated by a moving principle distinct from the grand moving principles, or maintaining powers, of the machines which they regulate. The pendulum is actuated by its own Gravity, and the balance by the balance-spring.

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The powers of the compounded machines, which are formed by various combinations of these Mechanic Powers, may be computed by compounding the ratios, which the power should have to the resistance, if each part were separately employed. But machines are generally less advantageous, as they are more complicated, since the defects of construction and other resistances are increased with the number of the parts. Various plans have been proposed for constructing machines, which should continue for ever any motion which had been once impressed. These schemes have accordingly been called plans for a Perpetual Motion. It is obvious, that no engine whose motion is derived from the continued operation of an external force, is entitled to this name. By it can only with propriety be understood a machine so contrived that, being once put in motion, it will continue that motion undiminished. The obstructions caused in every machine by the friction of its parts and the resistance of the air, seem to render such a plan impracticable; since it would be necessary that the impressed power should be continually increased by the machine, to compensate the continual diminutions, which it must sustain from the operation of these causes.

